

FORM TP 2011092

MAY/JUNE 2011

CARIBBEAN EXAMINATIONS COUNCIL SECONDARY EDUCATION CERTIFICATE EXAMINATION MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

18 MAY 2011 (a.m.)

INSTRUCTIONS TO CANDIDATES

- 1. This paper consists of **TWO** sections.
- 2. There are EIGHT questions in Section I and THREE questions in Section II.
- 3. Answer **ALL** questions in Section I, and any **TWO** questions from Section II.
- 4. Write your answers in the booklet provided.
- 5. All working must be clearly shown.
- 6. A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic calculator Geometry set Graph paper (provided)



Opposite

LIST OF FORMULAE

| Volume of a prism | V = | = Ah where A is the area of a cross-section and h is the | perpendicular |
|-------------------|-----|--|---------------|
| | 1 | -1 | |

length.

Volume of cylinder $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

 $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height. Volume of a right pyramid

 $C = 2\pi r$ where r is the radius of the circle. Circumference

Arc length
$$S = \frac{\theta}{360} \times 2\pi r$$
 where θ is the angle subtended by the arc, measured in

degrees.

 $A = \pi r^2$ where r is the radius of the circle. Area of a circle

Area of a sector
$$A = \frac{\theta}{360} \times \pi r^2$$
 where θ is the angle of the sector, measured in degrees.

Area of trapezium
$$A = \frac{1}{2}(a+b)h$$
 where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

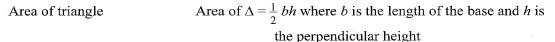
 $If ax^2 + bx + c = 0,$ Roots of quadratic equations

then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric ratios
$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

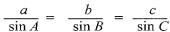
$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of
$$\triangle ABC = \frac{1}{2}ab \sin C$$

Area of
$$\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

where
$$s = \frac{a+b+c}{2}$$



Cosine rule
$$a^2 = b^2 + c^2 - 2bc \cos A$$



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Adjacent

Sine rule

SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, determine the EXACT value of:

(i)
$$\frac{2\frac{1}{4} + 1\frac{1}{8}}{4\frac{1}{2}}$$
 expressing your answer as a fraction (3 marks)

(ii)
$$3.96 \times 0.25 - \sqrt{0.0256}$$
 (3 marks)

(b) The table below shows Pamela's shopping bill. Some of the information was not included.

| Items | Quantity | Unit Price \$ | Total Cost \$ |
|-----------|-------------------|------------------|------------------|
| Rice | $6\frac{1}{2}$ kg | 2.40 | W |
| Potatocs | 4 bags | X | 52.80 |
| Milk | Y cartons | 2.35 | 14.10 |
| Sub Total | | | 82.50 |
| Z % VAT | | | 9.90 |
| TOTAL | | | 92.40 |

Calculate the values of W, X, Y and Z.

(5 marks)

2. (a) Write as a single fraction in its lowest terms

$$\frac{x-2}{3} + \frac{x+1}{4}$$
 (3 marks)

(b) The binary operation * is defined by

$$a * b = (a+b)^2 - 2ab.$$

Calculate the value of 3 * 4.

(2 marks)

(c) Factorise completely

(i)
$$xy^3 + x^2y$$
 (2 marks)

(ii)
$$2mh - 2nh - 3mk + 3nk$$
. (2 marks)

(d) The table below shows corresponding values of the variables x and y, where y varies directly as x.

| x | 2 | 5 | b |
|---|----|---|----|
| y | 12 | а | 48 |

Calculate the values of a and b.

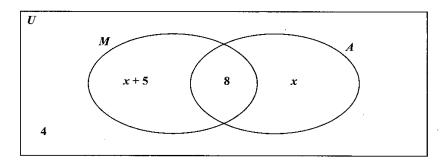
(3 marks)

3. (a) The Venn diagram below shows the number of students who study Music and Art in a class of 35 students.

 $U = \{\text{students in the class}\}\$

 $M = \{\text{students who study Music}\}\$

 $A = \{ \text{students who study Art} \}$



- (i) How many students study neither Art nor Music?
- (1 mark)

(ii) Calculate the value of x.

- (3 marks)
- (iii) Hence, state the number of students who study Music only.
- (1 mark)
- (b) (i) Using a ruler, pencil, a pair of compasses and a protractor, draw accurately a quadrilateral *EFGH* using the following measurements:

$$EF = 8 \text{ cm}$$
 $< EFG = 125^{\circ}$ $FG = 4 \text{ cm}$ $< HEF = 70^{\circ}$ $EH = 7 \text{ cm}$

(5 marks)

(ii) Measure and state in centimetres, the length of GH.

(1 mark)

4. (a) (i) Solve the inequality: 5-2x < 9

(2 marks)

- (ii) If x is an integer, determine the SMALLEST value of x that satisfies the inequality in (a) (i) above. (1 mark)
- (b) In this question, use $\pi = \frac{22}{7}$.
 - (i) A piece of wire is bent to form a square of area 121 cm².

Calculate:

- a) The length of each side of the square
- b) The perimeter of the square

(3 marks)

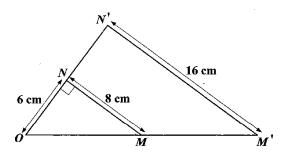
(ii) The same piece of wire is bent to form a circle.

Calculate:

- a) The radius of the circle
- b) The area of the circle

(4 marks)

5. (a) The diagram below, **not drawn to scale**, shows \triangle *OMN*, and its image, \triangle *OM'N'* under an enlargement with centre, O, and scale factor, k. Angle $ONM = 90^{\circ}$.



Using the dimensions shown on the diagram, calculate

(i) the value of k, the scale factor of the enlargement

(1 mark)

(ii) the length of OM

(1 mark)

(iii) the length of OM'.

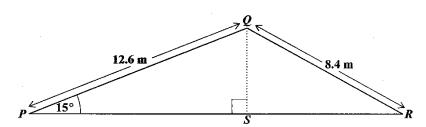
(2 marks)

(b) The diagram below, not drawn to scale, shows $\triangle PQR$, which represents the cross section of a roof. QS is perpendicular to PSR.

$$PQ = 12.6$$
 metres

$$QR = 8.4 \text{ metres}$$

$$\angle QPR = 15^{\circ}$$



Using the dimensions shown on the diagram, calculate, correct to 3 significant figures

(i) the length of QS

(2 marks)

(ii) the measure of $\angle RQS$

(3 marks)

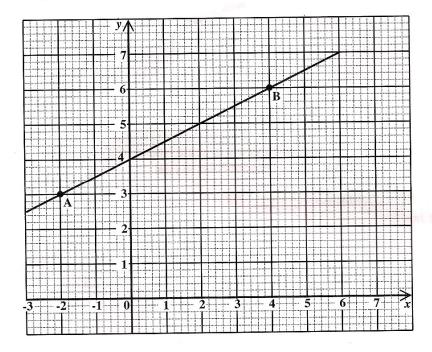
(iii) the area of $\triangle PQR$.

(3 marks)

6. (a) The functions f and g are defined by

$$f(x) = 6x + 8$$
 ; $g(x) = \frac{x-2}{3}$.

- (i) Calculate the value of $g\left(\frac{1}{2}\right)$. (2 marks)
- (ii) Write an expression for g f(x) in its simplest form. (2 marks)
- (iii) Find the inverse function $f^{-1}(x)$. (2 marks)
- (b) The diagram below shows the line segment which passes through the points A and B.



Determine

(i) the coordinates of A and B

(2 marks)

(ii) the gradient of the line segment AB

- (2 marks)
- (iii) the equation of the line which passes through A and B.
- (2 marks)

7. The table below shows the distribution of the masses of 100 packages.

| Mass (kg) | No. of Packages | Cumulative Frequency |
|-----------|-----------------|----------------------|
| 1 -10 | 12 | 12 |
| 11 – 20 | 28 | 40 |
| 21 – 30 | 30 | |
| 31 – 40 | 22 | , |
| 41 – 50 | 8 | |

- (a) Copy and complete the table to show the cumulative frequency for the distribution.

 (2 marks)
- (b) Using a scale of 2 cm to represent 10 kg on the x-axis and 1 cm to represent 10 packages on the y-axis, draw the cumulative frequency curve for the data.

(5 marks)

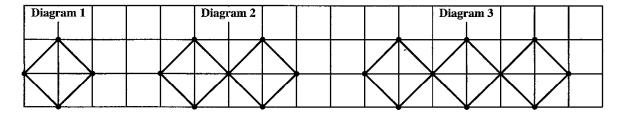
- (c) Estimate from the graph
 - (i) the median mass of the packages

(2 marks)

(ii) the probability that a package, chosen at random, has a mass which is LESS than 35 kg. (3 marks)

8. An answer sheet is provided for this question.

The figure below shows the first three diagrams in a sequence. Each diagram is made up of sticks joined at the ends by thumb tacks. The sticks are represented by lines and the thumb tacks by dots. In each diagram, there are t thumb tacks and s sticks.



On the answer sheet provided:

(a) Draw the FOURTH diagram in the sequence.

(2 marks)

(b) (i) How many sticks are in the SIXTH diagram?

(1 mark)

(ii) How many thumb tacks are in the SEVENTH diagram?

(1 mark)

(c) Complete the table by inserting the missing values at the rows marked (i) and (ii).

| | No. of Sticks | Rule Connecting t and s | No. of Thumb Tacks | |
|---|---------------|--|--------------------|-----------|
| | 4 | $1+\left(\frac{3}{4}\times 4\right)$ | 4 | |
| | 8 | $1+\left(\frac{3}{4}\times 8\right)$ | 7 | |
| | 12 | $1 + \left(\frac{3}{4} \times 12\right)$ | 10 | |
|) | 52 | | | (2 marks |
|) | | | 55 | (2 marks |

(d) Write the rule, in terms of s and t, to show how t is related to s.

(2 marks)

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Solve the pair of simultaneous equations

$$y = x^2 - x + 3$$

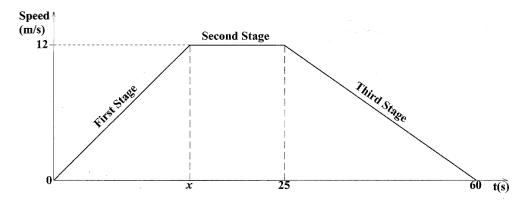
 $y = 6 - 3x$ (5 marks)

(b) Express the function $f(x) = 4x^2 - 8x - 2$ in the form $a(x+h)^2 + k$, where a, h and k are constants. (2 marks)

State

- (ii) the minimum value of f(x) (1 mark)
- (iii) the value of x for which f(x) is a minimum. (1 mark)

(c) The speed-time graph below, **not drawn to scale**, shows the three-stage journey of a racing car over a period of 60 seconds.



During the FIRST stage of the journey, the car increased its speed from 0 m/s to 12 m/s in x seconds accelerating at 0.6 m/s².

- (i) Calculate the value of x. (2 marks)
- (ii) What is the gradient of the graph during the SECOND stage? Explain, in one sentence, what the car is doing during this stage. (2 marks)
- (iii) Calculate the distance travelled by the car on the THIRD stage of the journey.

 (2 marks)

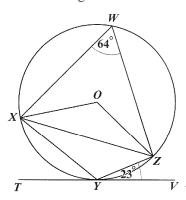
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Total 15 marks

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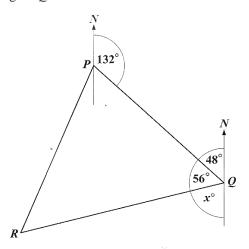
MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) In the diagram below, **not drawn to scale**, W, X, Y and Z are points on the circumference of a circle, centre O. TYV is a tangent to the circle at Y, $\angle XWZ = 64^{\circ}$ and $\angle ZYV = 23^{\circ}$.



Calculate, giving reasons for your answer, the measure of angle

- (i) XYZ (2 marks)
- (ii) YXZ (2 marks)
- (iii) OXZ. (3 marks)
- (b) The diagram below, **not drawn to scale**, shows the route of an aeroplane flying from Portcity (P) to Queenstown (Q) and then to Riversdale (R). The bearing of Q from P is 132° and the angle PQR is 56° .



- (i) Calculate the value of x, as shown in the diagram.
- (2 marks)
- (ii) The distance from Portcity (P) to Queenstown (Q) is 220 kilometres and the distance from Queenstown to Riversdale (R) is 360 kilometres. Calculate the distance RP.
- (iii) Determine the bearing of R from P.

(3 marks)

Total 15 marks

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VECTORS AND MATRICES

- 11. (a) Determine the inverse of the matrix $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$. (2 marks)
 - (b) The transformation, $M = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$, maps the points R and T onto R' and T' such that:

$$R(7,2) \longrightarrow R'(2,-7)$$
 and

$$T(-5, 4) \longrightarrow T'(4, 5)$$

(i) Determine the values of a and b.

(2 marks)

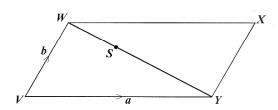
(ii) Describe fully the transformation, M.

(3 marks)

(c) WXYV is a parallelogram in which

$$\overrightarrow{VY} = a$$
 and $\overrightarrow{VW} = b$.

S is a point on WY such that WS: SY = 1:2.



- (i) Write in terms of a and b, an expression for:
 - a) \overrightarrow{WY}
 - b) \overrightarrow{WS}
 - c) \overrightarrow{SX}

(5 marks)

(ii) R is the mid-point of VW. Prove that R, S and X are collinear.

(3 marks)

Total 15 marks

END OF TEST