FORM TP 2010087

TEST CODE **01234020**

MAY/JUNE 2010

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

19 MAY 2010 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. This paper consists of **TWO** sections.
2. There are EIGHT questions in Section I and THREE questions in Section II.
3. Answer **ALL** questions in Section I, and any **TWO** questions from Section II.
4. Write your answers in the booklet provided.
5. All working must be clearly shown.
6. A list of formulae is provided on page 2 of this booklet.

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Required Examination Materials

Electronic calculator
Geometry set
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Volume of a prism	$V = Ah$ where A is the area of a cross-section and h is the perpendicular length.
Volume of cylinder	$V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.
Volume of a right pyramid	$V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.
Circumference	$C = 2\pi r$ where r is the radius of the circle.
Area of a circle	$A = \pi r^2$ where r is the radius of the circle.
Area of trapezium	$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations

If $ax^2 + bx + c = 0$,

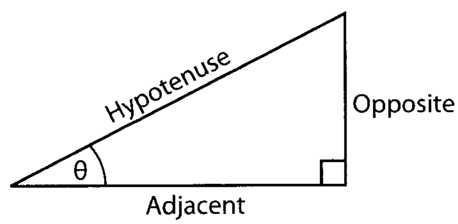
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$



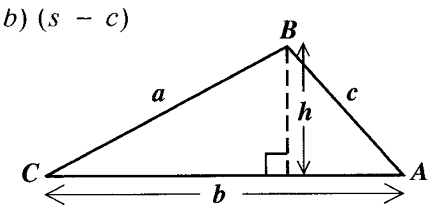
Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a + b + c}{2}$



Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

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SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Determine the EXACT value of:

(i) $$\frac{1\frac{1}{2} - \frac{2}{5}}{4\frac{2}{5} \times \frac{3}{4}}$$ **(3 marks)**

(ii) $2.5^2 - \dfrac{2.89}{17}$

giving your answer correct to 2 significant figures **(3 marks)**

(b) Mrs. Jack bought 150 T-shirts for \$1 920 from a factory.

(i) Calculate the cost of ONE T-shirt. **(1 mark)**

The T-shirts are sold at \$19.99 each.

Calculate

(ii) the amount of money Mrs. Jack received after selling ALL of the T-shirts **(1 mark)**

(iii) the TOTAL profit made **(1 mark)**

(iv) the profit made as a percentage of the cost price, giving your answer correct to the nearest whole number. **(2 marks)**

Total 11 marks

2. (a) Given that $a = -1$, $b = 2$ and $c = -3$, find the value of:

(i) $a + b + c$ **(1 mark)**

(ii) $b^2 - c^2$ **(1 mark)**

(b) Write the following phrases as algebraic expressions:

(i) seven times the sum of x and y **(1 mark)**

(ii) the product of TWO consecutive numbers when the smaller number is y **(1 mark)**

(c) Solve the pair of simultaneous equations:

$2x + y = 7$
$x - 2y = 1$ **(3 marks)**

(d) Factorise completely:

(i) $4y^2 - z^2$ **(1 mark)**

(ii) $2ax - 2ay - bx + by$ **(2 marks)**

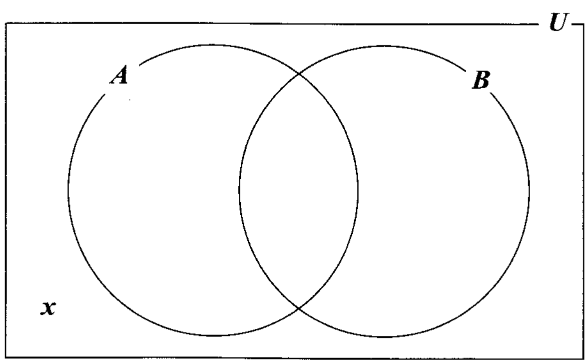
(iii) $3x^2 + 10x - 8$ **(2 marks)**

Total 12 marks

3. (a) A survey was conducted among 40 tourists. The results were:

28 visited Antigua (A)
30 visited Barbados (B)
$3x$ visited both Antigua and Barbados
x visited neither Antigua nor Barbados

(i) Copy and complete the Venn diagram below to represent the given information above.

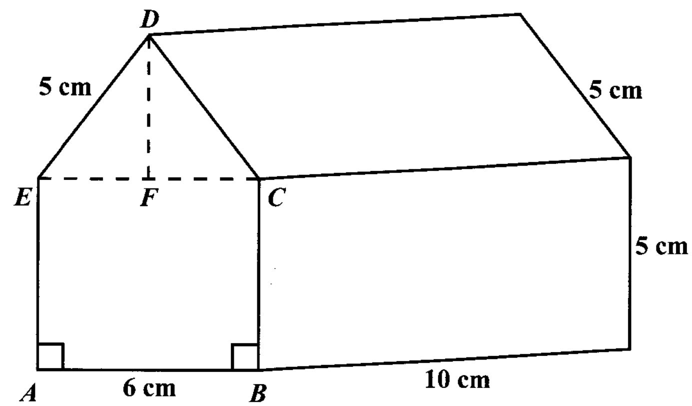


(2 marks)

(ii) Write an expression, in x, to represent the TOTAL number of tourists in the survey. **(2 marks)**

(iii) Calculate the value of x. **(2 marks)**

(b) The diagram below, **not drawn to scale**, shows a wooden toy in the shape of a prism, with cross section $ABCDE$. F is the midpoint of EC, and $\angle BAE = \angle CBA = 90°$.



Calculate

(i) the length of EF **(1 mark)**

(ii) the length of DF **(2 marks)**

(iii) the area of the face $ABCDE$. **(3 marks)**

Total 12 marks

4. (a) When y varies directly as the square of x, the variation equation is written $y = kx^2$, where k is a constant.

(i) Given that $y = 50$ when $x = 10$, find the value of k. **(2 marks)**

(ii) Calculate the value of y when $x = 30$. **(2 marks)**

(b) (i) **Using a ruler, a pencil and a pair of compasses**, construct triangle EFG with
$EG = 6$ cm
$\angle FEG = 60°$
and $\angle EGF = 90°$. **(5 marks)**

(ii) Measure and state

a) the length of EF

b) the size of $\angle EFG$. **(2 marks)**

Total 11 marks

5. (a) The functions f and g are defined as $f(x) = 2x - 5$ and $g(x) = x^2 + 3$.

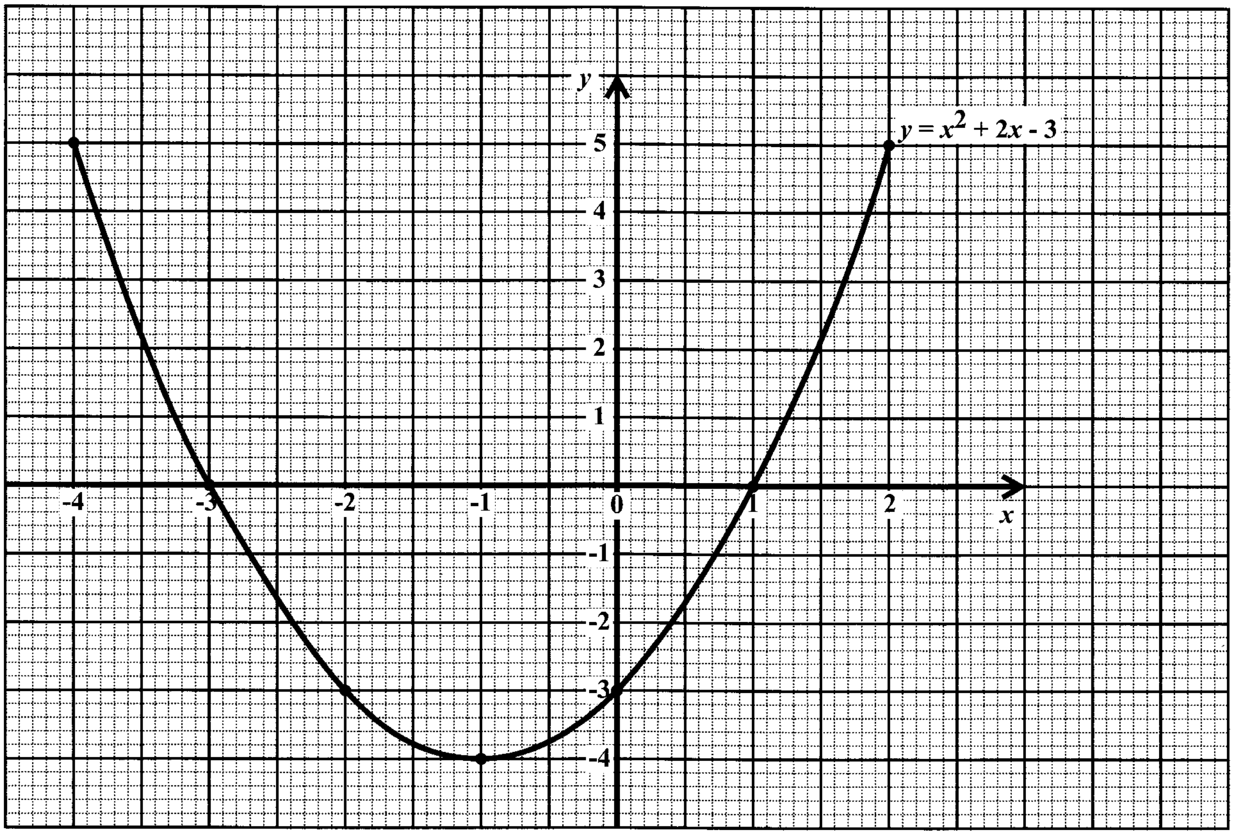
(i) Calculate the value of

a) $f(4)$ **(1 mark)**

b) $gf(4)$. **(2 marks)**

(ii) Find $f^{-1}(x)$. **(2 marks)**

(b) The diagram below shows the graph of $y = x^2 + 2x - 3$ for the domain $-4 \leq x \leq 2$.



Use the graph above to determine

(i) the scale used on the x-axis **(1 mark)**

(ii) the value of y for which $x = -1.5$ **(2 marks)**

(iii) the values of x for which $y = 0$ **(2 marks)**

(iv) the range of values of y, giving your answer in the form $a \leq y \leq b$, where a and b are real numbers. **(2 marks)**

Total 12 marks

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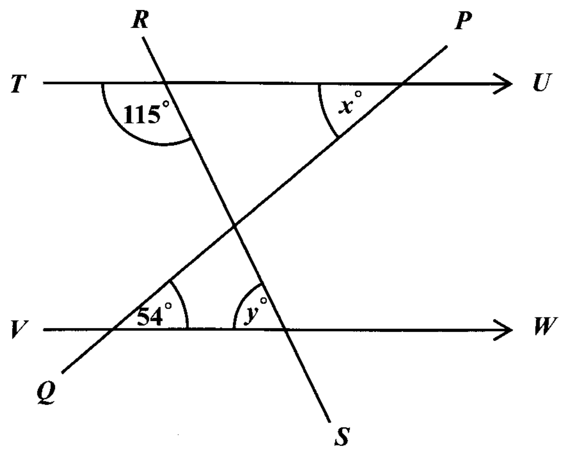
6. An answer sheet is provided for this question.

(a) The diagram below, **not drawn to scale**, shows two straight lines, PQ and RS, intersecting a pair of parallel lines, TU and VW.

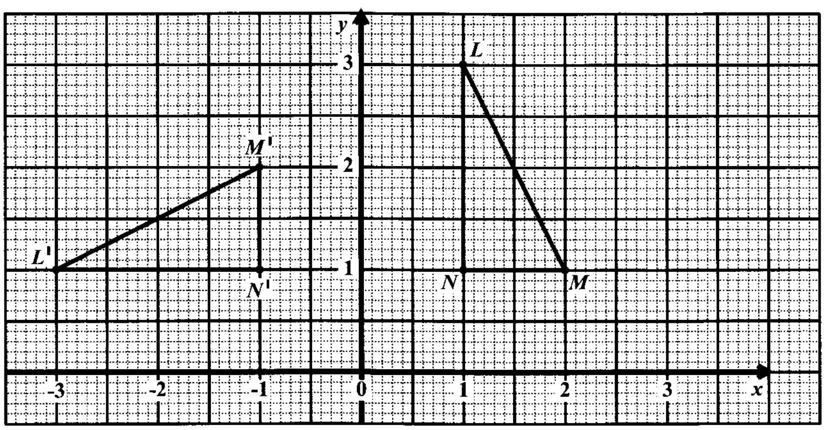


Determine, giving a reason for EACH of your answers, the value of

(i) x **(2 marks)**

(ii) y. **(2 marks)**

(b) The diagram below shows triangle LMN, and its image, triangle $L'M'N'$, after undergoing a rotation.



(i) Describe the rotation FULLY by stating

a) the centre

b) the angle

c) the direction. **(3 marks)**

(ii) State TWO geometric relationships between triangle LMN and its image, triangle $L'M'N'$. **(2 marks)**

(iii) Triangle LMN is translated by the vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

Determine the coordinates of the image of the point L under this transformation. **(2 marks)**

Total 11 marks

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7. A class of 24 students threw the cricket ball at sports. The distance thrown by each student was measured to the nearest metre. The results are shown below.

22	50	35	52	47	30
48	34	45	23	43	40
55	29	46	56	43	59
36	63	54	32	49	60

(a) Copy and complete the frequency table for the data shown above.

Distance (m)	**Frequency**
20 – 29	3
30 – 39	5
______	______
______	______
______	______

(**3 marks**)

(b) State the lower boundary for the class interval 20 – 29. (**1 mark**)

(c) Using a scale of **1 cm on the x-axis to represent 5 metres**, and a scale of **1 cm on the y-axis to represent 1 student**, draw a histogram to illustrate the data.

(**5 marks**)

(d) Determine

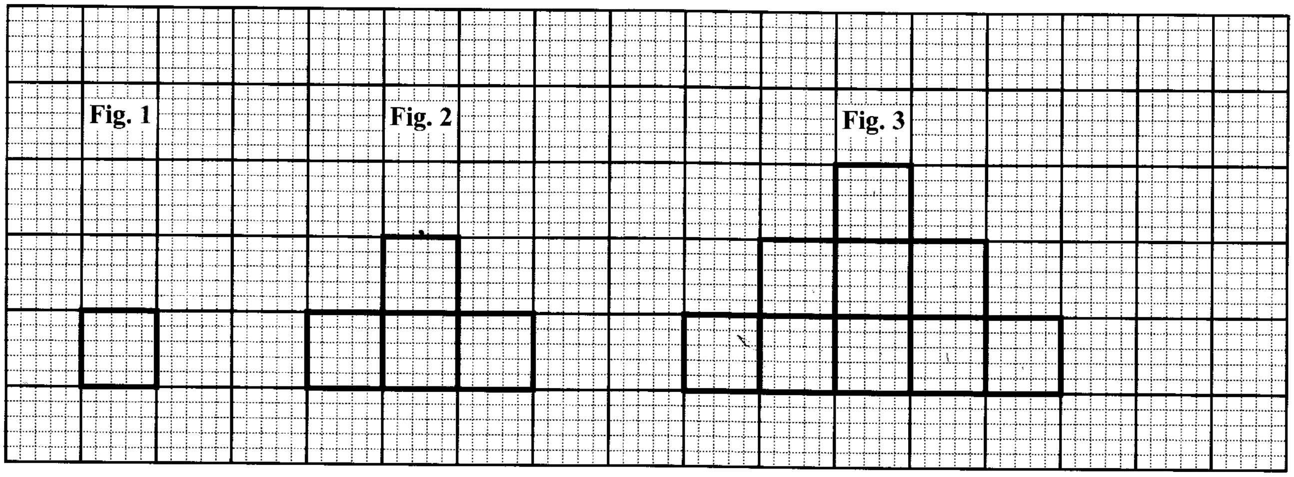
(i) the number of students who threw the ball a distance recorded as 50 metres or more (**1 mark**)

(ii) the probability that a student, chosen at random, threw the ball a distance recorded as 50 metres or more. (**1 mark**)

Total 11 marks

8. **An answer sheet is provided for this question.**

The diagram below shows the first three figures in a sequence of figures. Each figure is made up of squares of side 1 cm.



(a) **On your answer sheet**, draw the FOURTH figure (Fig. 4) in the sequence.

(2 marks)

(b) Study the patterns in the table shown below, and **on the answer sheet provided**, complete the rows numbered (i), (ii), (iii) and (iv).

	Figure	Area of Figure (cm^2)	Perimeter of Figure (cm)	
	1	1	$1 \times 6 - 2 = 4$	
	2	4	$2 \times 6 - 2 = 10$	
	3	9	$3 \times 6 - 2 = 16$	
(i)	4	_____	__________	**(2 marks)**
(ii)	5	_____	__________	**(2 marks)**
(iii)	15	_____	__________	**(2 marks)**
(iv)	n	_____	__________	**(2 marks)**

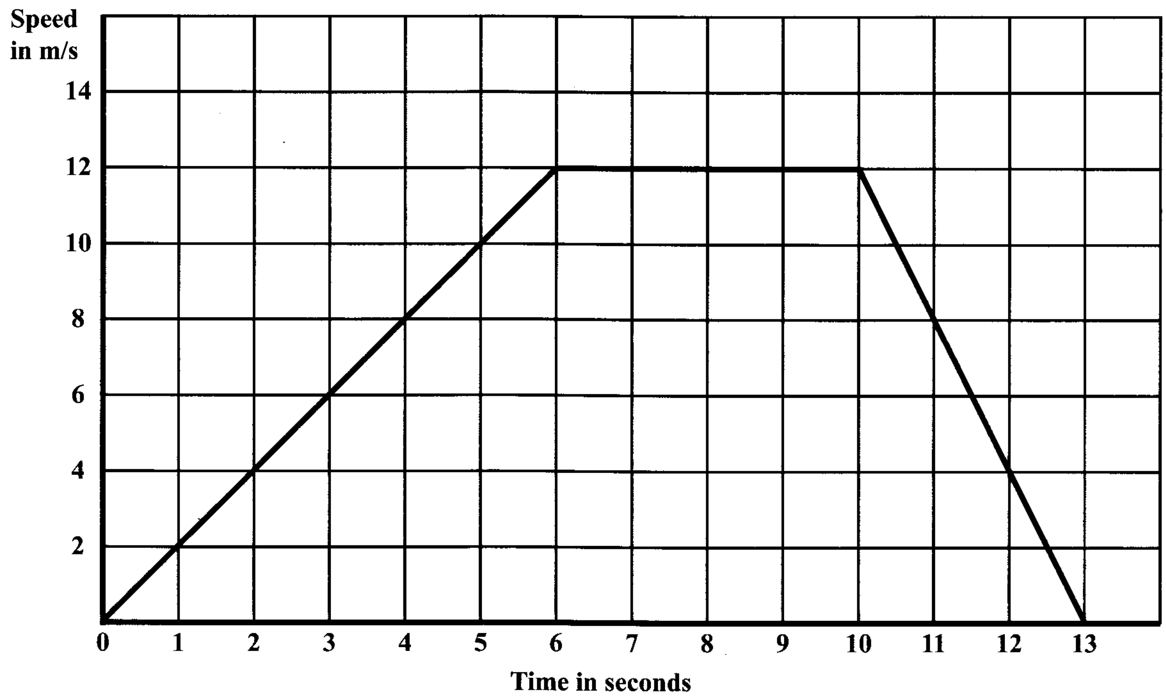
Total 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The diagram below shows the speed-time graph of the motion of an athlete during a race.



(i) **Using the graph**, determine

a) the MAXIMUM speed

b) the number of seconds for which the speed was constant

c) the TOTAL distance covered by the athlete during the race.

(4 marks)

(ii) During which time-period of the race was

a) the speed of the athlete increasing

b) the speed of the athlete decreasing

c) the acceleration of the athlete zero? **(3 marks)**

(b) A farmer supplies his neighbours with x pumpkins and y melons daily, using the following conditions:

First condition : $y \geq 3$
Second condition : $y \leq x$
Third condition : the total number of pumpkins and melons must not exceed 12.

(i) Write an inequality to represent the THIRD condition. **(1 mark)**

(ii) Using a scale of **1 cm to represent one pumpkin** on the x-axis and **1 cm to represent one melon** on the y-axis, draw the graphs of the THREE lines associated with the THREE inequalities. **(4 marks)**

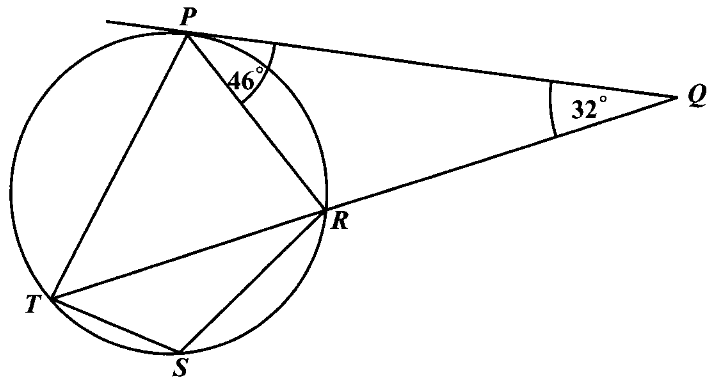
(iii) Identify, by shading, the region which satisfies the THREE inequalities. **(1 mark)**

(iv) Determine, from your graph, the **minimum** values of x and y which satisfy the conditions. **(2 marks)**

Total 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) In the diagram below, **not drawn to scale**, PQ is a tangent to the circle $PTSR$, so that
$\angle RPQ = 46°$
$\angle RQP = 32°$
and TRQ is a straight line.



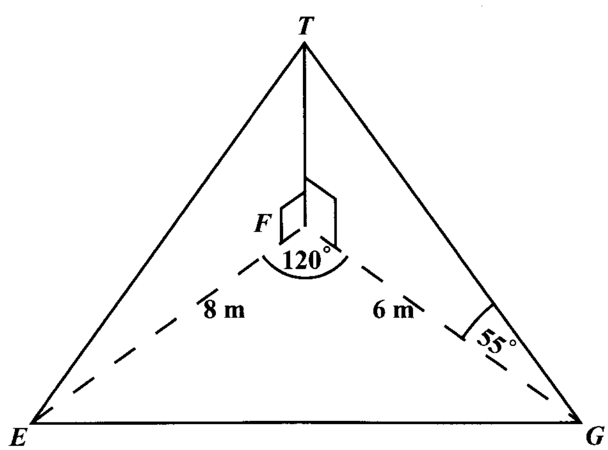
Calculate, giving a reason for EACH step of your answer,

(i) $\angle PTR$ **(2 marks)**

(ii) $\angle TPR$ **(3 marks)**

(iii) $\angle TSR$. **(2 marks)**

(b) The diagram below, **not drawn to scale**, shows a vertical flagpole, FT, with its foot, F, on the horizontal plane EFG. ET and GT are wires which support the flagpole in its position. The angle of elevation of T from G is 55°, EF = 8 m, FG = 6 m and $\angle EFG = 120°$.



Calculate, giving your answer correct to 3 significant figures

(i) the height, FT, of the flagpole **(2 marks)**

(ii) the length of EG **(3 marks)**

(iii) the angle of elevation of T from E. **(3 marks)**

Total 15 marks

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VECTORS AND MATRICES

11. (a) **A** and **B** are two 2×2 matrices such that

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}.$$

(i) Find **AB**. **(2 marks)**

(ii) Determine $\mathbf{B}^{-1}$, the inverse of **B**. **(1 mark)**

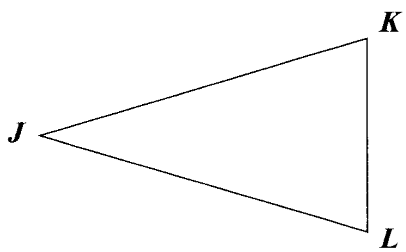
(iii) Given that

$$\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix},$$

write $\begin{pmatrix} x \\ y \end{pmatrix}$ as the product of TWO matrices. **(2 marks)**

(iv) Hence, calculate the values of x and y. **(2 marks)**

(b) The diagram below, **not drawn to scale**, shows triangle JKL.



M and N are points on JK and JL respectively, such that

$$JM = \frac{1}{3}JK \quad \text{and} \quad JN = \frac{1}{3}JL.$$

(i) Copy the diagram in your answer booklet and show the points M and N. **(2 marks)**

(ii) Given that $\overrightarrow{JM} = \boldsymbol{u}$ and $\overrightarrow{JN} = \boldsymbol{v}$,
write, in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$, an expression for

a) $\overrightarrow{JK}$

b) $\overrightarrow{MN}$

c) $\overrightarrow{KL}$. **(4 marks)**

(iii) Using your findings in (b) (ii), deduce TWO geometrical relationships between KL and MN. **(2 marks)**

Total 15 marks

END OF TEST