

**CARIBBEAN EXAMINATIONS COUNCIL
SECONDARY EDUCATION CERTIFICATE
EXAMINATION
MATHEMATICS**

Paper 02 – General Proficiency

2 hours 40 minutes

20 MAY 2009 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables
Graph paper (provided)



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SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

$$(i) \frac{2\frac{2}{3} + 1\frac{1}{5}}{6\frac{2}{5}}$$

giving your answer as a common fraction

(3 marks)

$$(ii) \sqrt{\left(\frac{0.0256}{25}\right)}$$

giving your answer in standard form.

(3 marks)

- (b) The basic wage earned by a truck driver for a 40-hour week is \$560.00.

- (i) Calculate his hourly rate.

(1 mark)

For overtime work, the driver is paid one and a half times the basic hourly rate.

- (ii) Calculate his overtime wage for 10 hours of overtime.

(2 marks)

- (iii) Calculate the TOTAL wages earned by the truck driver for a 55-hour week.

(3 marks)

Total 12 marks

2. (a) Factorise completely:

$$(i) 2ax + 3ay - 2bx - 3by$$

(2 marks)

$$(ii) 5x^2 - 20$$

(2 marks)

$$(iii) 3x^2 + 4x - 15$$

(2 marks)

- (b) One packet of biscuits costs \$ x and one cup of ice cream costs \$ y .

One packet of biscuits and two cups of ice cream cost \$8.00, while three packets of biscuits and one cup of ice cream cost \$9.00.

- (i) Write a pair of simultaneous equations in x and y to represent the given information above.

(2 marks)

- (ii) Solve the equations obtained in (b) (i) above to find the cost of one packet of biscuits and the cost of one cup of ice cream.

(4 marks)

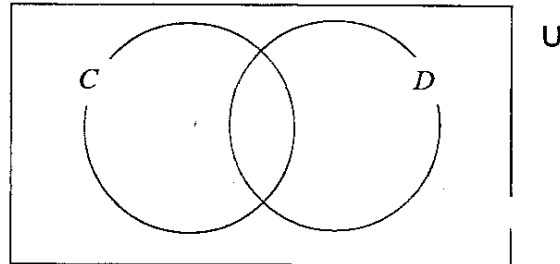
Total 12 marks

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3. (a) In a survey of 50 students,
 23 owned cellular phones
 18 owned digital cameras
 x owned cellular phones and digital cameras
 $2x$ owned neither.

Let C represent the set of students in the survey who owned cellular phones, and D the set of students who owned digital cameras.

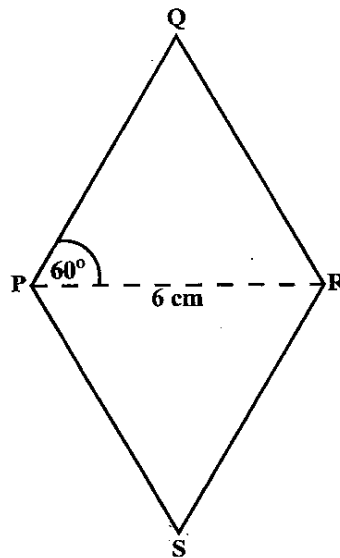
- (i) Copy and complete the Venn diagram below to represent the information obtained from the survey.



(2 marks)

- (ii) Write an expression in x for the TOTAL number of students in the survey. (1 mark)
- (iii) Calculate the value of x . (2 marks)

- (b) The diagram below, **not drawn to scale**, shows a rhombus, $PQRS$, with the diagonal $PR = 6$ cm, and the angle $RPQ = 60^\circ$.



- (i) Using a ruler, a pencil, and a pair of compasses, construct the rhombus $PQRS$ accurately. (4 marks)
- (ii) Join QS . Measure and state, in centimetres, the length of QS . (2 marks)

Total 11 marks

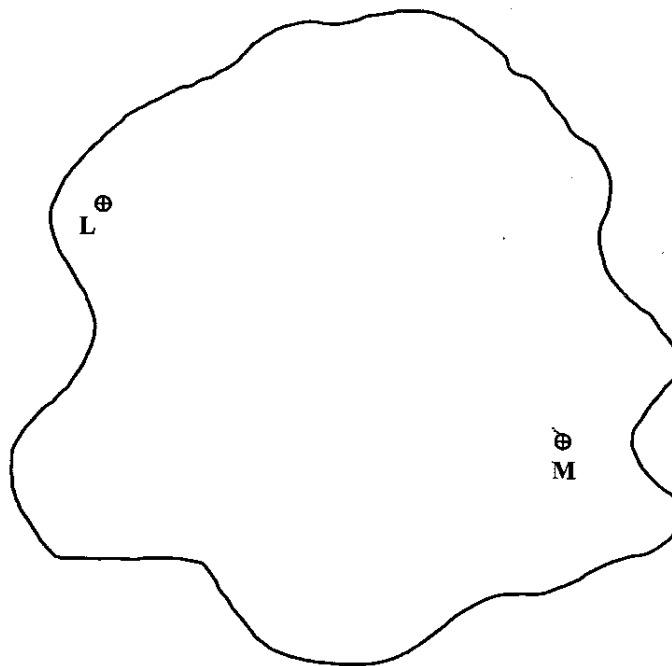
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4. (a) The table below shows two readings taken from an aircraft's flight record.

Time	Distance Travelled (km)
08:55	957
09:07	1083

For the period of time between the two readings, calculate

- (i) the distance travelled in kilometres (1 mark)
- (ii) the average speed of the aircraft in km/h. (3 marks)
- (b) The map shown below is drawn to a scale of 1:50 000.



- (i) Measure and state, in centimetres, the distance on the map from *L* to *M* along a straight line. (2 marks)
- (ii) Calculate the actual distance, in kilometres, from *L* to *M*. (2 marks)
- (iii) The actual distance between two points is 4.5 km. Calculate the number of centimetres that should be used to represent this distance on the map. (3 marks)

Total 11 marks

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5. (a) Given that $f(x) = 2x - 5$ and $g(x) = x^2 - 31$, calculate the value of

(i) $f(-2)$ (1 mark)

(ii) $gf(1)$ (2 marks)

(iii) $f^{-1}(3)$. (2 marks)

(b) Given that $y = x^2 + 2x - 3$

(i) Copy and complete the table below.

x	-4	-3	-2	-1	0	1	2
y	5		-3	-4	-3		5

(2 marks)

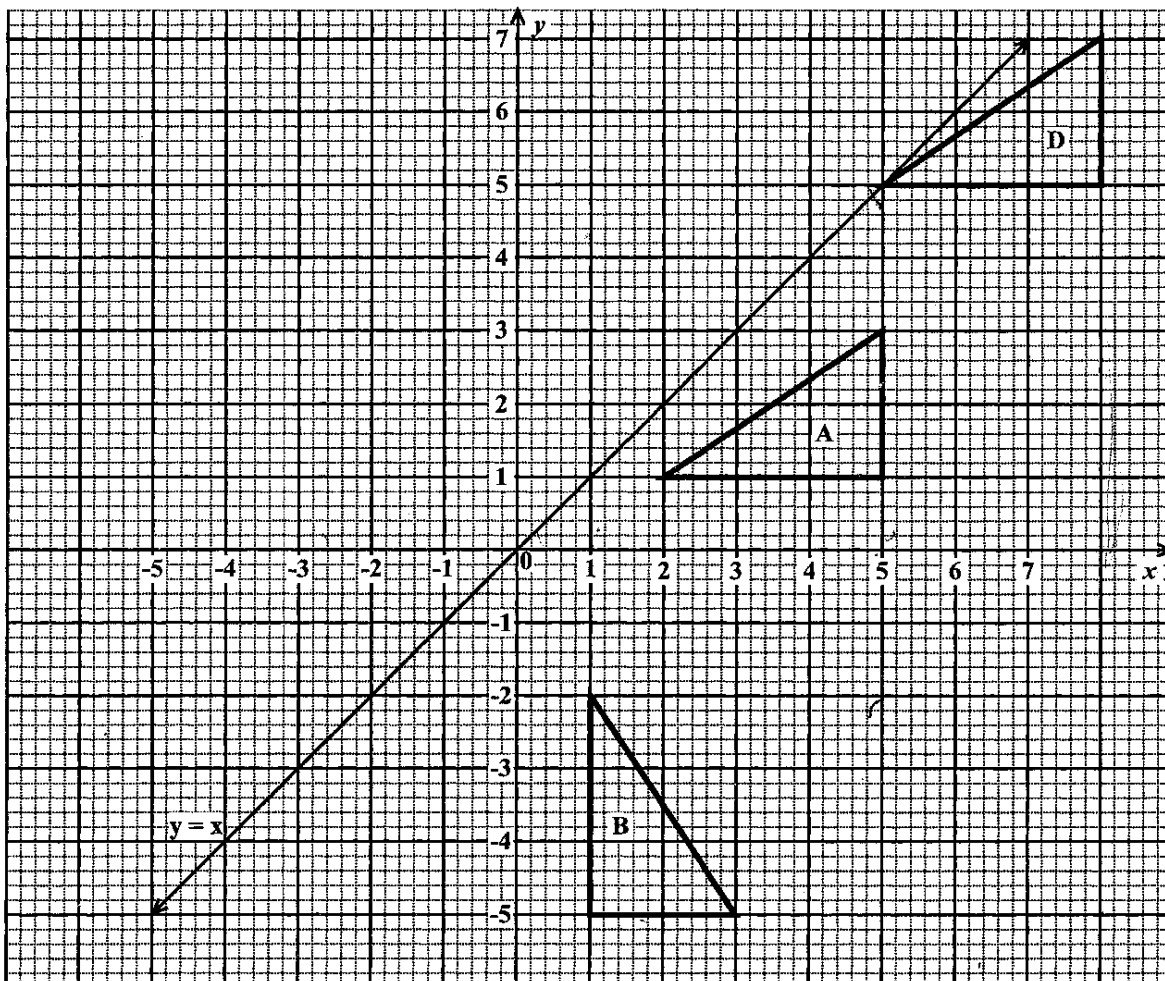
(ii) Using a scale of **2 cm to represent 1 unit on the x-axis** and **1 cm to represent 1 unit on the y-axis**, draw the graph of $y = x^2 + 2x - 3$ for $-4 \leq x \leq 2$.

(5 marks)

Total 12 marks

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6. The diagram below shows triangles A , B and D . The line $y = x$ is also shown.



- (a) Describe, FULLY, the single transformation which maps triangle A onto
- triangle D (3 marks)
 - triangle B . (3 marks)
- (b) State the coordinates of the vertices of triangle C , the image of triangle A after a reflection in the line $y = x$. (4 marks)

Total 10 marks

A

7. **An answer sheet is provided for this question.**

The table below shows the time, to the nearest minute, that 80 students waited to be served at a school's canteen.

Waiting Time (minutes)	No. of Students	Cumulative Frequency
1 – 5	4	4
6 – 10	7	11
11 – 15	11	22
16 – 20	18	
21 – 25	22	
26 – 30	10	
31 – 35	5	
36 – 40	3	

- (a) Copy and complete the table, showing the cumulative frequency. (2 marks)
- (b) **On the answer sheet provided**, use the values from your table to complete the cumulative frequency curve. (4 marks)
- (c) **Use your graph from (b) above** to estimate
- (i) the median for the data (2 marks)
- (ii) the number of students who waited for **no more than** 29 minutes (2 marks)
- (iii) the probability that a student, chosen at random from the group, waited for **no more than** 17 minutes. (2 marks)

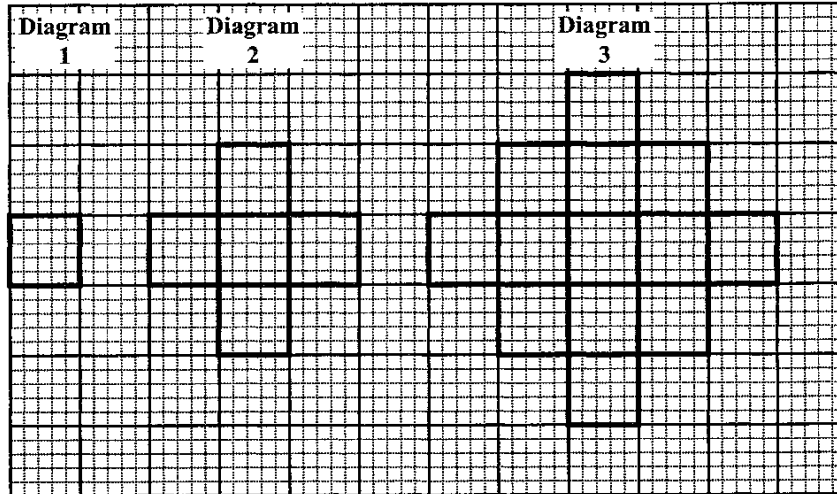
Total 12 marks

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8. An answer sheet is provided for this question:

The drawings below show the first three diagrams in a sequence. Each diagram in the sequence is obtained by drawing a 1-unit square on each side that forms the perimeter of the previous diagram.

For example, Diagram 2 is obtained by drawing a 1-unit square on each of the four sides of Diagram 1.



On the answer sheet provided:

- (a) Draw Diagram 4 in the sequence. (2 marks)
- (b) Complete the table by inserting the appropriate values at the rows marked (i), (ii) and (iii). (8 marks)

Total 10 marks

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SECTION II**Answer TWO questions in this section.****RELATIONS, FUNCTIONS AND GRAPHS**

9. (a) Solve the pair of simultaneous equations
- $$y = 4 - 2x$$
- $$y = 2x^2 - 3x + 1. \quad (4 \text{ marks})$$
- (b) Express $2x^2 - 3x + 1$ in the form $a(x + h)^2 + k$, where a , h and k are real numbers. **(3 marks)**
- (c) Using your answer from (b) above, or otherwise, calculate
- (i) the minimum value of $2x^2 - 3x + 1$ **(1 mark)**
- (ii) the value of x for which the minimum occurs. **(1 mark)**
- (d) Sketch the graph of $y = 2x^2 - 3x + 1$, clearly showing
- the coordinates of the minimum point
 - the value of the y -intercept
 - the values of x where the graph cuts the x -axis. **(4 marks)**
- (e) Sketch on your graph of $y = 2x^2 - 3x + 1$, the line which intersects the curve at the values of x and y calculated in (a) above. **(2 marks)**

Total 15 marks

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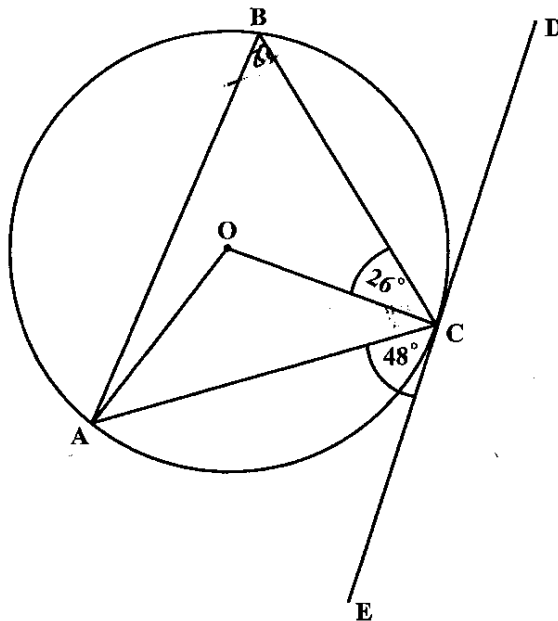
10. (a) The owner of a shop wishes to buy x guitars and y violins. To satisfy the demands of his customers, the number of violins must be less than or equal to the number of guitars.
- (i) Write an inequality to represent this information. (1 mark)
- The cost of one guitar is \$150 and the cost of one violin is \$300. He has \$4 500 to spend on the purchase of these instruments.
- (ii) Write an inequality to represent this information. (2 marks)
- To get a good bargain, the owner of the shop must buy at least 5 violins.
- (iii) Write an inequality to represent this information. (1 mark)
- (b) (i) Using a scale of **2 cm on the horizontal axis to represent 5 guitars**, and **2 cm on the vertical axis to represent 5 violins**, draw the graphs of the lines associated with the **THREE** inequalities written in (a) (i), (ii) and (iii) above. (4 marks)
- (ii) Shade the region on your graph that satisfies all **THREE** inequalities. (1 mark)
- (iii) State the coordinates of the vertices of the shaded region. (2 marks)
- (c) The owner of the shop sells the instruments to make a profit of \$60 on each guitar and \$100 on each violin.
- (i) Express the **TOTAL** profit in terms of x and y . (1 mark)
- (ii) Calculate the **maximum** profit. (3 marks)

Total 15 marks

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GEOMETRY AND TRIGONOMETRY

11. (a) The diagram below, **not drawn to scale**, shows a circle, centre O . The line DCE is a tangent to the circle. Angle $ACE = 48^\circ$ and angle $OCB = 26^\circ$.

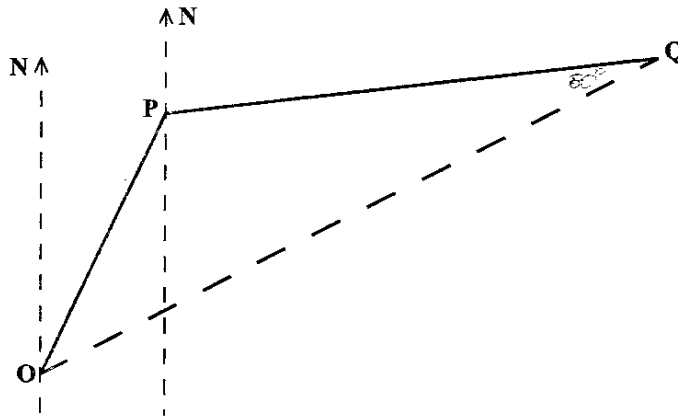


Calculate:

- | | | |
|-------|--------------|------------|
| (i) | $\angle ABC$ | (1 mark) |
| (ii) | $\angle AOC$ | (1 mark) |
| (iii) | $\angle BCD$ | (1 mark) |
| (iv) | $\angle BAC$ | (1 mark) |
| (v) | $\angle OAC$ | (1 mark) |
| (vi) | $\angle OAB$ | (1 mark) |

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- (b) The diagram below, **not drawn to scale**, shows the positions of two hurricane tracking stations, P and Q , relative to a point O . P is on a bearing of 025° from O , and $OP = 400$ km. Q is on a bearing of 080° from P and $PQ = 700$ km.



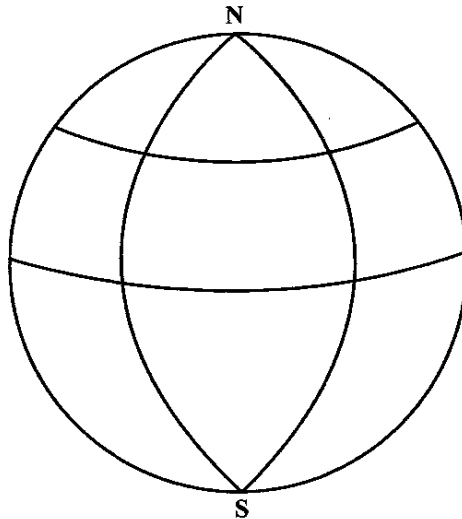
- (i) Copy the diagram above. On your diagram label the angles that show the bearings of 025° and 080° . (2 marks)
- (ii) Calculate
- $\angle OPQ$
 - the length, to the nearest kilometre, of OQ
 - the bearing of Q from O . (7 marks)

Total 15 marks

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12. In this question, use $\pi = \frac{22}{7}$, and assume that the earth is a sphere of radius 6370 km.

The diagram below, **not drawn to scale**, shows a sketch of the earth with the North and South Poles labelled N and S respectively.



Arcs representing circles of longitude 35°E and 15°W , and circles of latitude 0° and 60°N are drawn but not labelled.

- (a) Copy the sketch and
- (i) label the arc which represents:
 - a) 60°N
 - b) 35°E (2 marks)
 - (ii) insert the points:
 - a) J (60°N , 35°E)
 - b) K (60°N , 15°W) (2 marks)
- (b) Calculate, **to the nearest kilometre**, the SHORTEST distance from
- (i) J to the North Pole measured along the common circle of longitude (3 marks)
 - (ii) J to K measured along the common circle of latitude. (4 marks)
- (c) A point H is located 2002 km due south of K along the common circle of longitude.
Calculate the latitude of H. (4 marks)

Total 15 marks

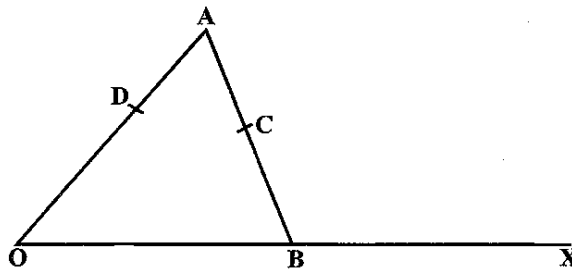
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VECTORS AND MATRICES

13. (a) The points A and B have position vectors $\vec{OA} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ where O is the origin $(0, 0)$. The point G lies on the line AB such that $AG = \frac{1}{3}AB$.

Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

- (i) the vectors \vec{AB} and \vec{AG} (4 marks)
- (ii) the position vector \vec{OG} . (2 marks)
- (b) In the diagram below, **not drawn to scale**, B is the midpoint of OX , C is the midpoint of AB , and D is such that $OD = 2DA$. The vectors \mathbf{a} and \mathbf{b} are such that $\vec{OA} = 3\mathbf{a}$ and $\vec{OB} = \mathbf{b}$.



- (i) Write in terms of \mathbf{a} and \mathbf{b} :
- a) \vec{AB}
- b) \vec{AC}
- c) \vec{DC}
- d) \vec{DX} (6 marks)
- (ii) State TWO geometrical relationships between DX and DC . (2 marks)
- (iii) State ONE geometrical relationship between the points D , C , and X . (1 mark)

Total 15 marks

14. (a) The value of the determinant of $M = \begin{pmatrix} x & 4 \\ 3 & x \end{pmatrix}$ is 13.

Calculate the values of x .

(4 marks)

- (b) The transformation R is represented by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

The transformation S is represented by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

- (i) Write a single matrix, in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ to represent the combined transformation S followed by R . (2 marks)

- (ii) Calculate the image of the point $(5, -2)$ under the combined transformation in (b) (i) above. (3 marks)

- (c) The matrix $N = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$.

- (i) Determine the inverse matrix of N . (2 marks)

- (ii) Hence, calculate the value of x and the value of y for which

$$3x - y = 5$$

$$2x + 5y = 9.$$

(4 marks)

Total 15 marks

END OF TEST