TEST CODE **01234020**

FORM TP 2008100

MAY/JUNE 2008

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

21 MAY 2008 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of trapezium — $A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

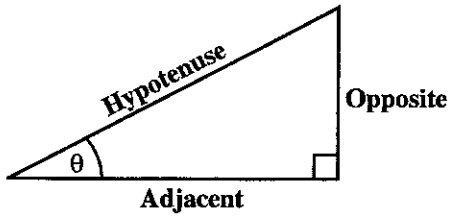
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

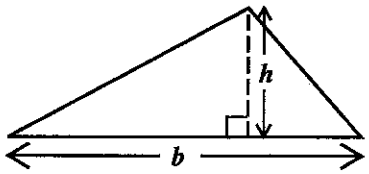
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

Area of $\Delta ABC = \frac{1}{2}ab \sin C$

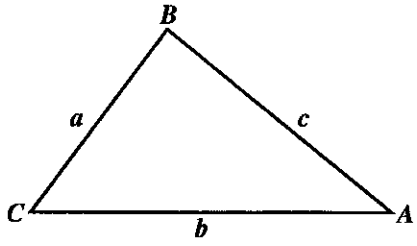


Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a + b + c}{2}$

Sine rule — $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$



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SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) (i) Using a calculator, or otherwise, determine the EXACT value of

$$(3.9 \times 0.27) + \sqrt{0.6724}.$$

(2 marks)

(ii) Express as a single fraction

$$\frac{2\frac{1}{2} - \frac{4}{5}}{\frac{3}{4}}.$$

(3 marks)

(b) In this question, use CAN \$1.00 = JA \$72.50.

(i) On a vacation in Canada, Steve used his credit card to buy a camera for CAN \$250.00.

What is the value of the camera in Jamaican dollars? **(2 marks)**

(ii) Steve's credit card limit is JA \$30 000.00. After buying the camera, how many Canadian dollars does he have left on his credit card for spending?

(3 marks)

Total 10 marks

2. (a) Find the value of EACH of the following when $a = 2, b = -1, c = 3$

(i) $a(b + c)$ **(1 mark)**

(ii) $\dfrac{4b^2 - 2ac}{a+b+c}$. **(2 marks)**

(b) Change the following statements into algebraic expressions:

(i) Four times the sum of x and 5 **(1 mark)**

(ii) 16 larger than the product of a and b **(2 marks)**

(c) Solve the equation

$15 - 4x = 2(3x + 1)$. **(2 marks)**

(d) Factorise completely

(i) $6a^2b^3 + 12a^4b$ **(2 marks)**

(ii) $2m^2 + 9m - 5$. **(2 marks)**

Total 12 marks

3. At a career guidance seminar, a survey was done to find out the type of careers that Form 5 students were likely to choose.

The results are shown in the table below.

Career	Lawyer	Teacher	Doctor	Artist	Salesperson
Number of students	240	189	t	216	330

There were 1 080 students surveyed.

(a) Calculate the value of t, the number of students who were interested in becoming doctors. **(2 marks)**

(b) (i) The data above are to be represented on a pie chart. Calculate the size of the angle in each sector of the pie chart. **(4 marks)**

(ii) Using a circle of radius 4 cm, construct the pie chart to represent the data. **(4 marks)**

Total 10 marks

4. (a) A Universal set, U, is defined as
$U = \{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$.

Sets M and N are subsets of U such that
M = {Prime Numbers} and N = {Even Numbers}.

(i) Draw a Venn diagram to represent the sets M, N and U. **(5 marks)**

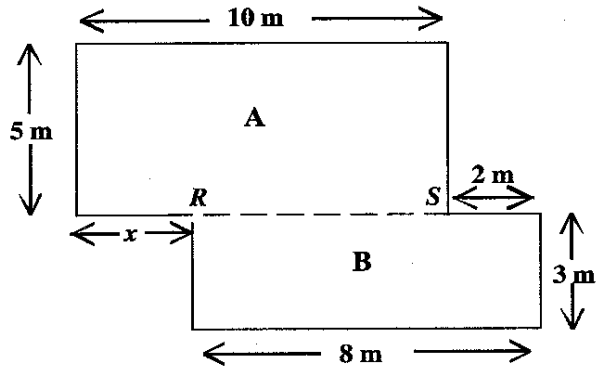
(ii) List the elements of the set $(M \cup N)'$. **(1 mark)**

(b) (i) **Using only a pair of compasses, a ruler and a pencil**, construct parallelogram *ABCD* in which $AB = AD = 7$ cm and the angle *BAD* is 60°. **(5 marks)**

(ii) Measure and write down the length of the diagonal *AC*. **(1 mark)**

Total 12 marks

5. The diagram below, **not drawn to scale**, represents the plan of a floor.
The broken line *RS*, divides the floor into two rectangles, **A** and **B**.



(a) (i) Calculate the length of *RS*. **(1 mark)**

(ii) Hence state the value of *x*. **(1 mark)**

(b) Calculate the perimeter of the entire floor. **(3 marks)**

(c) Calculate the area of the entire floor. **(3 marks)**

(d) Section **A** of the floor is to be covered with flooring boards measuring 1 **m** by 20 **cm**.
How many flooring boards are needed for covering Section **A**? **(4 marks)**

Total 12 marks

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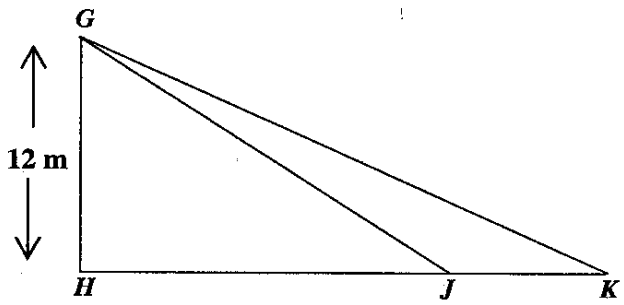
6. (a) In the diagram below, **not drawn to scale**, GH is a vertical pole standing on a horizontal plane and H, J and K are points on the horizontal plane.



$GH = 12$ metres and the angles of elevation of the top of the pole G from J and K are 32° and 27° respectively.

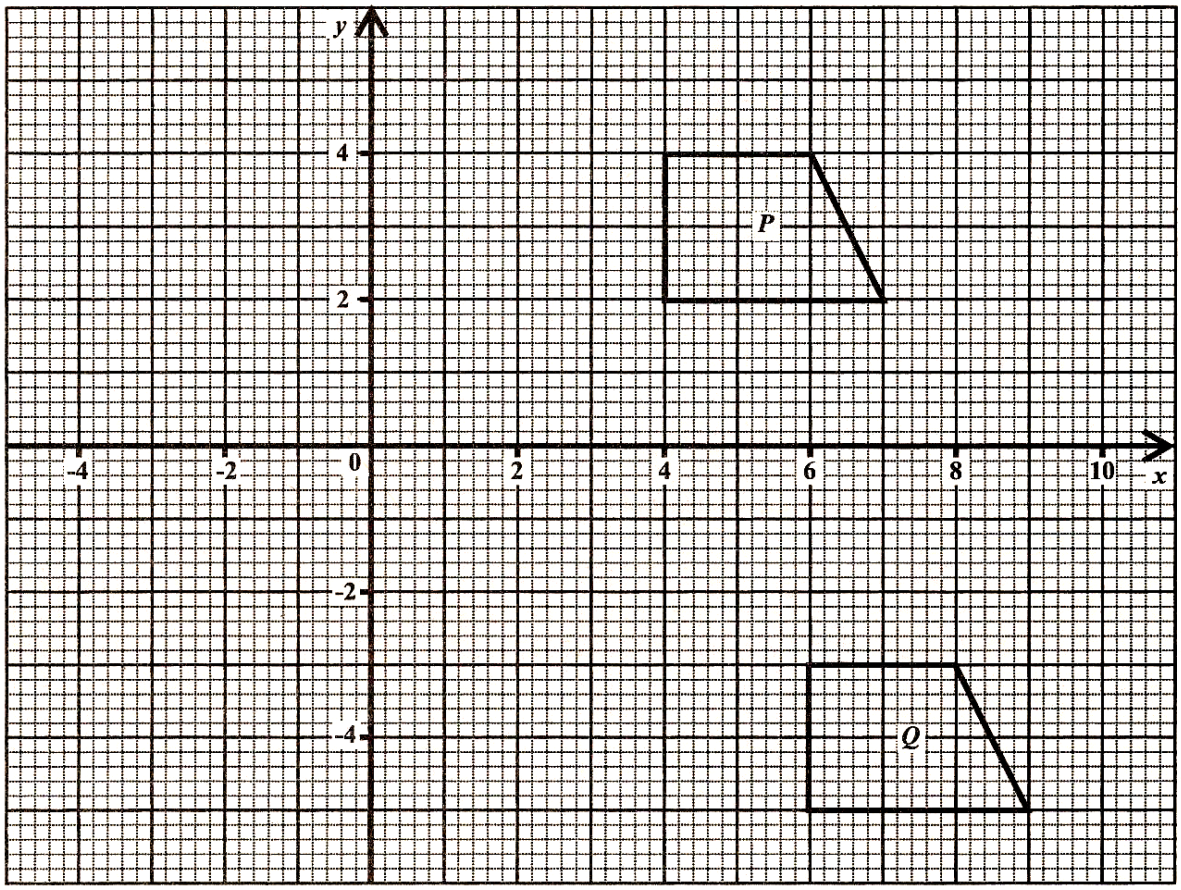
(i) Copy the diagram and insert the angles of elevation. **(1 mark)**

(ii) Calculate to **one decimal place**

a) the length of HJ

b) the length of JK. **(5 marks)**

(b) **An answer sheet is provided for this question.**



(i) The figure labelled P undergoes a transformation, such that its image is Q. Describe this transformation completely. **(2 marks)**

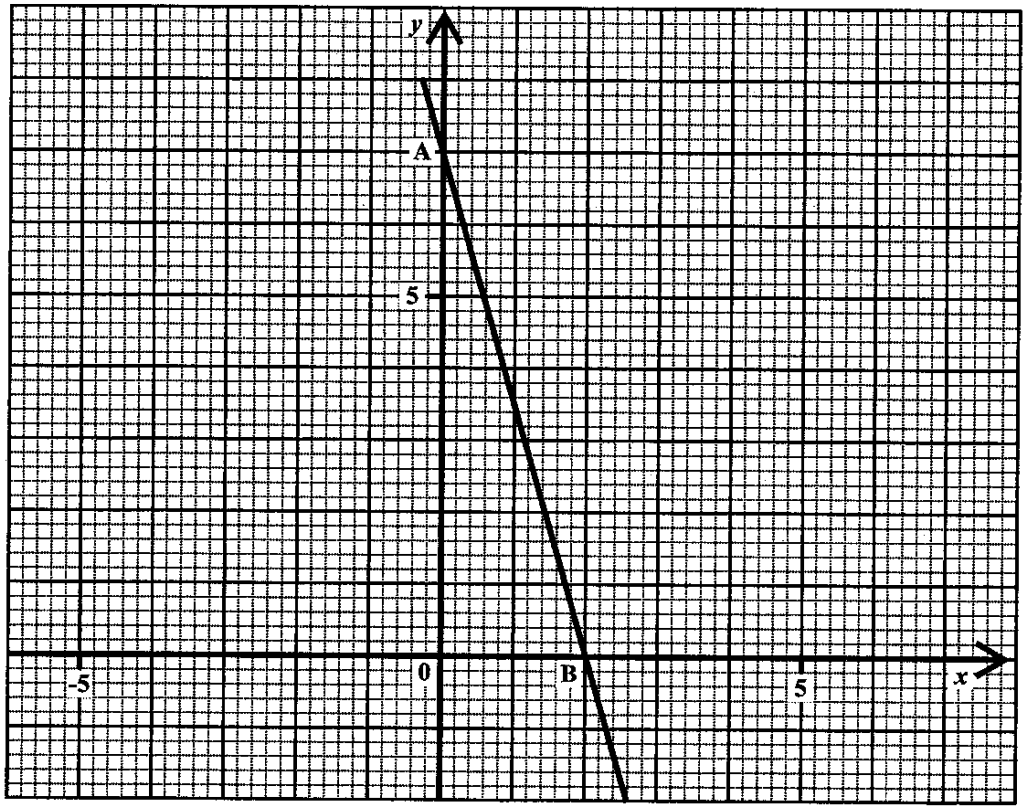
(ii) **On the answer sheet provided**, draw and label

a) the line $y = x$

b) S, the image of P under a reflection in the line $y = x$. **(4 marks)**

Total 12 marks

7. The diagram below shows the graph of a straight line passing through the points A and B.



(a) The equation of the line above is $y = mx + c$.

(i) State the value of c. **(1 mark)**

(ii) Determine the value of m. **(2 marks)**

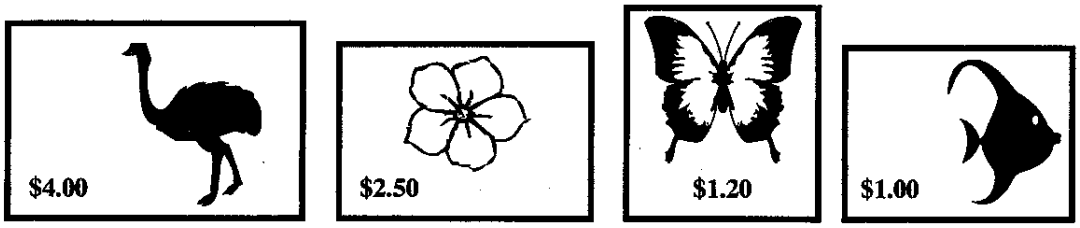
(iii) Determine the coordinates of the mid-point of the line segment AB. **(2 marks)**

(b) The point $(-2, k)$ lies on the line. Determine the value of k. **(3 marks)**

(c) Determine the coordinates of the point of intersection of the line $y = x - 2$ and the line shown above. **(4 marks)**

Total 12 marks

8. Annie went to the post office and bought a collection of SIX of each of the following stamps.



(a) What was the TOTAL cost of the stamps? **(2 marks)**

(b) She had to post a parcel and the total cost of postage was $25.70. What stamps can she select from the collection, to make up this amount if she must use

(i) as many $4.00 stamps as possible? **(3 marks)**

(ii) all her $1.00 stamps? **(2 marks)**

(c) (i) What is the LARGEST number of stamps that she can use from the collection to post the parcel?

(ii) List the selection of stamps she can use. **(3 marks)**

Total 10 marks

SECTION II

Answer TWO questions in this section.

RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Simplify

(i) $x^2 \times x^3 \div x^4$ **(1 mark)**

(ii) $a^{\frac{3}{2}}b^{\frac{5}{2}} \times \sqrt{ab^3}$. **(2 marks)**

(b) If $f(x) = 2x - 3$, find the value of

(i) $f(2)$ **(1 mark)**

(ii) $f^{-1}(0)$ **(2 marks)**

(iii) $f^{-1}f(2)$ **(2 marks)**

(c) The temperature, K, of a liquid t minutes after heating is given in the table below.

t **(time in minutes)**	0	10	20	30	40	50	60
K **(Temp. in °C)**	84	61	40	29	27	26	25

(i) Using a scale of 2 cm to represent 10 seconds on the horizontal axis and a scale of 2 cm to represent 10 degrees on the vertical axis, construct a temperature-time graph to show how the liquid cools in the 60 minute interval.

Draw a smooth curve through all the plotted points. **(4 marks)**

(ii) Use your graph to estimate

a) the temperature of the liquid after 15 minutes

b) the rate of cooling of the liquid at $t = 30$ minutes. **(3 marks)**

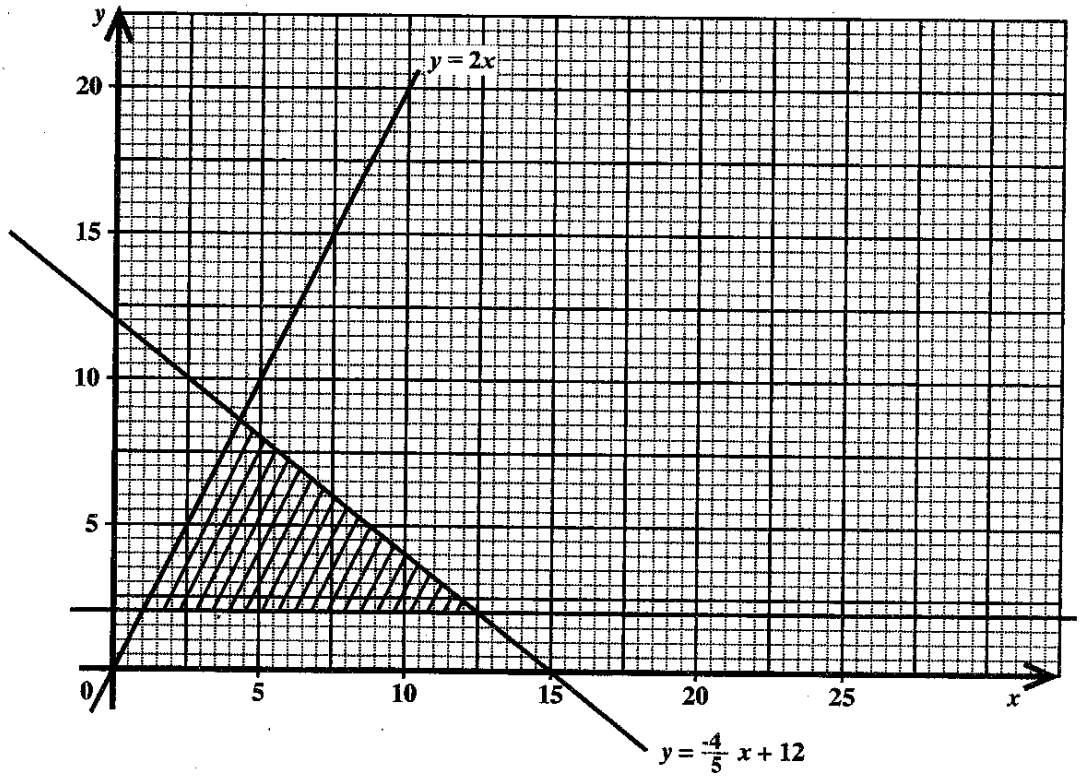
Total 15 marks

10. (a) Solve the following pair of equations for x and y:

$$y + 4x = 27$$
$$xy + x = 40$$

(6 marks)

(b) The shaded area in the diagram below shows the solution of a set of inequalities in x and y. The variable x represents the number of boys in a cricket club and y represents the number of girls in the cricket club.



Use the graph above to answer the questions which follow.

(i) State, **using arguments based on the graph**, whether the cricket club can have as members:

a) 10 boys and 5 girls

b) 6 boys and 6 girls. **(2 marks)**

(ii) Write down the set of THREE inequalities that define the shaded region. **(4 marks)**

(iii) A company sells uniforms for the club and makes a profit of \$3.00 on a boy's uniform and \$5.00 on a girl's uniform.

a) Write an expression in x and y that represents the total profit made by the company on the sale of uniforms.

b) Calculate the **minimum** profit the company can make. **(3 marks)**

Total 15 marks

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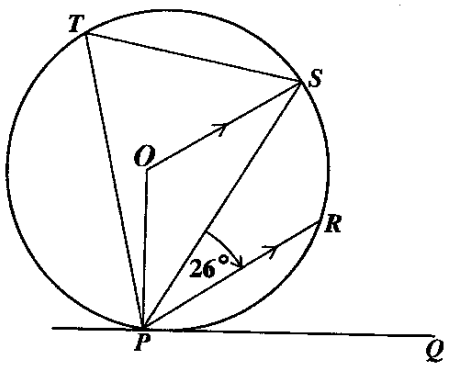
GEOMETRY AND TRIGONOMETRY

11. (a) In the diagram below, **not drawn to scale**, PQ is a tangent to the circle, centre O. PR is parallel to OS and angle $SPR = 26\,^{\circ}$.

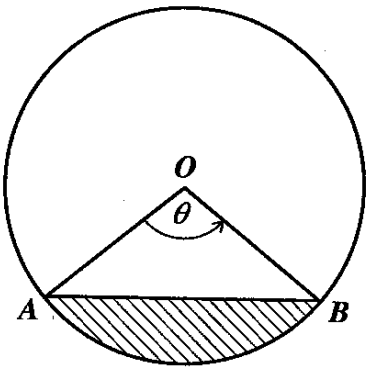


Calculate, **giving reasons for your answer**, the size of

(i) angle PTS **(2 marks)**

(ii) angle RPQ. **(2 marks)**

(b) In the diagram below, **not drawn to scale**, O is the centre of the circle of radius 8.5 cm and AB is a chord of length 14.5 cm.



(i) Calculate the value of θ **to the nearest degree.** **(3 marks)**

(ii) Calculate the area of triangle AOB. **(2 marks)**

(iii) Hence, calculate the area of the shaded region. [Use $\pi = 3.14$]. **(3 marks)**

(iv) Calculate the length of the major arc AB. **(3 marks)**

Total 15 marks

12. A ship leaves Port R, sails to Port S and then to Port T.

The bearing of S from R is 112°.

The bearing of T from S is 033°.

The distance RT is 75 km and the distance RS is 56 km.

(a) Draw a diagram showing the journey of the ship from R to S to T.
Show on your diagram

(i) the North direction **(1 mark)**

(ii) the bearings 112° and 033° **(2 marks)**

(iii) the points R, S and T **(1 mark)**

(iv) the distances 75 km and 56 km. **(1 mark)**

(b) Calculate

(i) the size of angle RST **(1 mark)**

(ii) the size of angle RTS **(3 marks)**

(iii) the bearing of R from T. **(2 marks)**

(c) The ship leaves Port T and travels due west to a point X which is due north of R.

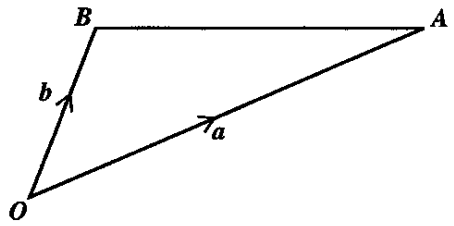
(i) Show on your diagram the journey from T to X. **(1 mark)**

(ii) Calculate the distance TX. **(3 marks)**

Total 15 marks

VECTORS AND MATRICES

13. The position vectors of A and B relative to the origin are $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively.



The point P is on OA such that $OP = 2PA$.
The point M is on BA such that $BM = MA$.

(a) Copy the diagram and complete it to show the points of P and M. **(2 marks)**

(b) OB is produced to N such that $OB = BN$.

(i) Show the position of N on your diagram. **(1 mark)**

(ii) Express in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ the vectors $\overrightarrow{AB}$, $\overrightarrow{PA}$ and $\overrightarrow{PM}$. **(5 marks)**

(c) Use a vector method to prove that P, M and N are collinear. **(4 marks)**

(d) Calculate the length of AN if

$$\boldsymbol{a} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

(3 marks)

Total 15 marks

14. (a) X and Y are two matrices where

$$X = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix} \text{ and } Y = \begin{pmatrix} 4 & -1 \\ 3 & 7 \end{pmatrix}.$$

Evaluate $X^2 + Y$. **(4 marks)**

(b) The matrix $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ maps $Q\,(1, 2)$ to $Q'(5, 7)$.

Find the 2 x 2 matrix which maps Q' back to Q. **(2 marks)**

(c) The vertices of triangle DEF are

$D(5, 12)$, $E(2, 7)$ and $F(8, 4)$.

(i) Triangle DEF undergoes an enlargement with centre, O, and scale factor, k. Its image is $D'E'F'$ where

$D\,(5, 12) \rightarrow D'(7.5, 18)$.

a) Determine the value of k.

b) Hence write down the coordinates of E' and F'. **(4 marks)**

(ii) $D'E'F'$ undergoes a clockwise rotation of 90° about the origin.

a) Determine the 2 x 2 matrix that represents a clockwise rotation of 90° about the origin.

b) Determine the coordinates of $D''E''F''$, the image of $D'E'F'$, under this rotation.

c) Determine the 2 x 2 matrix that maps triangle DEF onto triangle $D''E''F''$.
(5 marks)

Total 15 marks

END OF TEST