TEST CODE **01234020**

FORM TP 2008017

JANUARY 2008

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

04 JANUARY 2008 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

8017

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of trapezium — $A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

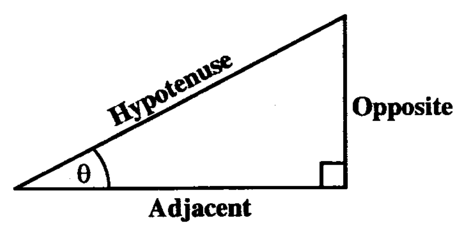
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

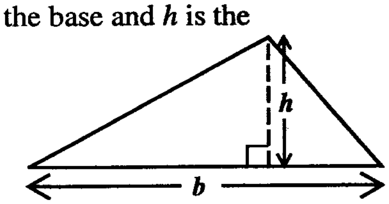
$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$



Area of triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height



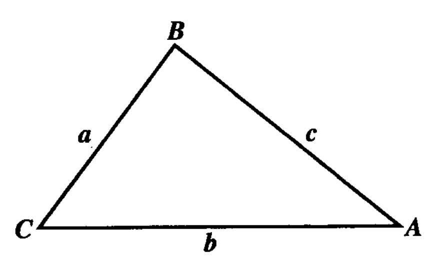
Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$

Sine rule — $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$



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SECTION I

Answer ALL the questions in this section.

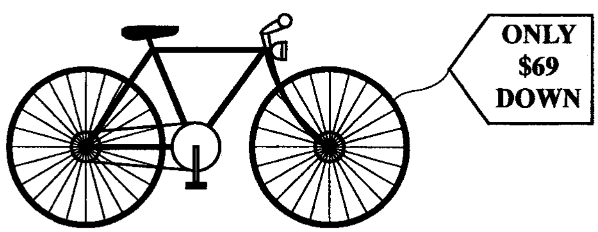
All working must be clearly shown.

1. (a) Calculate the EXACT value of:

(i) $\dfrac{1\frac{1}{7} - \frac{3}{4}}{2\frac{1}{2} \times \frac{1}{5}}$ (**4 marks**)

(ii) $2 - \dfrac{0.24}{0.15}$ (**2 marks**)

(b) The cash price of a bicycle is \$319.95. It can be bought on hire purchase by making a deposit of \$69.00 and 10 monthly installments of \$28.50 EACH.



(i) What is the TOTAL hire purchase price of the bicycle? (**2 marks**)

(ii) Calculate the difference between the total hire purchase price and the cash price. (**1 mark**)

(iii) Express your answer in (ii) above as a percentage of the cash price. (**2 marks**)

Total 11 marks

2. (a) (i) Solve the inequality: $3 - 2x < 7$ **(2 marks)**

(ii) If x is a whole number, determine the SMALLEST value that satisfies the inequality in (a) (i) above. **(1 mark)**

(b) Factorize completely

(i) $x^2 - xy$ **(1 mark)**

(ii) $a^2 - 1$ **(1 mark)**

(iii) $2p - 2q - p^2 + pq$ **(2 marks)**

(c) The table below shows the types of cakes available at a bakery, the cost of each cake and the number of cakes sold for a given day.

TYPE OF CAKE	COST ($)	NO. OF CAKES SOLD
Sponge	(k+5)	2
Chocolate	k	10
Fruit	2k	4

(i) Write an expression, in terms of k, for the amount of money collected from the sale of sponge cakes for the day. **(1 mark)**

(ii) Write an expression, in terms of k, for the TOTAL amount of money collected. **(2 marks)**

The total amount of money collected at the bakery for the day was $140.00.

(iii) Determine the value of k. **(2 marks)**

Total 12 marks

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3. (a) S and T are subsets of a Universal set U such that:

$U = \{k, l, m, n, p, q, r\}$
$S = \{k, l, m, p\}$
$T = \{k, p, q\}$

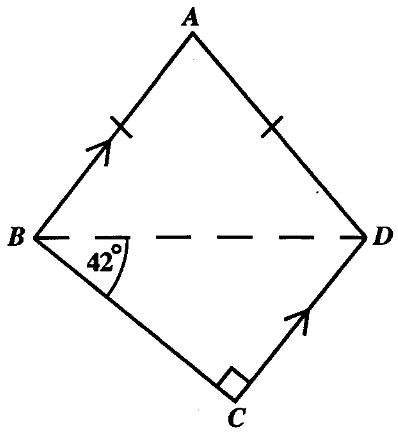
(i) Draw a Venn diagram to represent this information. **(3 marks)**

(ii) List, using set notation, the members of the set

a) $S \cup T$ **(1 mark)**

b) S' **(2 marks)**

(b) The diagram below, **not drawn to scale**, shows a quadrilateral ABCD with AB = AD, $\angle BCD = 90^\circ$ and $\angle DBC = 42^\circ$. AB is parallel to DC.



Calculate, giving reasons for your answers, the size of EACH of the following angles.

(i) $\angle ABC$

(ii) $\angle ABD$

(iii) $\angle BAD$. **(6 marks)**

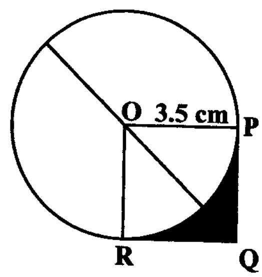
Total 12 marks

4. (a) John left Port A at 0730 hours and travels to Port B in the same time zone.

(i) He arrives at Port B at 1420 hours. How long did the journey take?
(1 mark)

(ii) John travelled 410 kilometres. Calculate his average speed in km h^{-1}.
(2 marks)

(b) The diagram below, **not drawn to scale**, shows a circle with centre O and a square OPQR. The radius of the circle is 3.5 cm.



Use $\pi = \frac{22}{7}$

Calculate the area of:

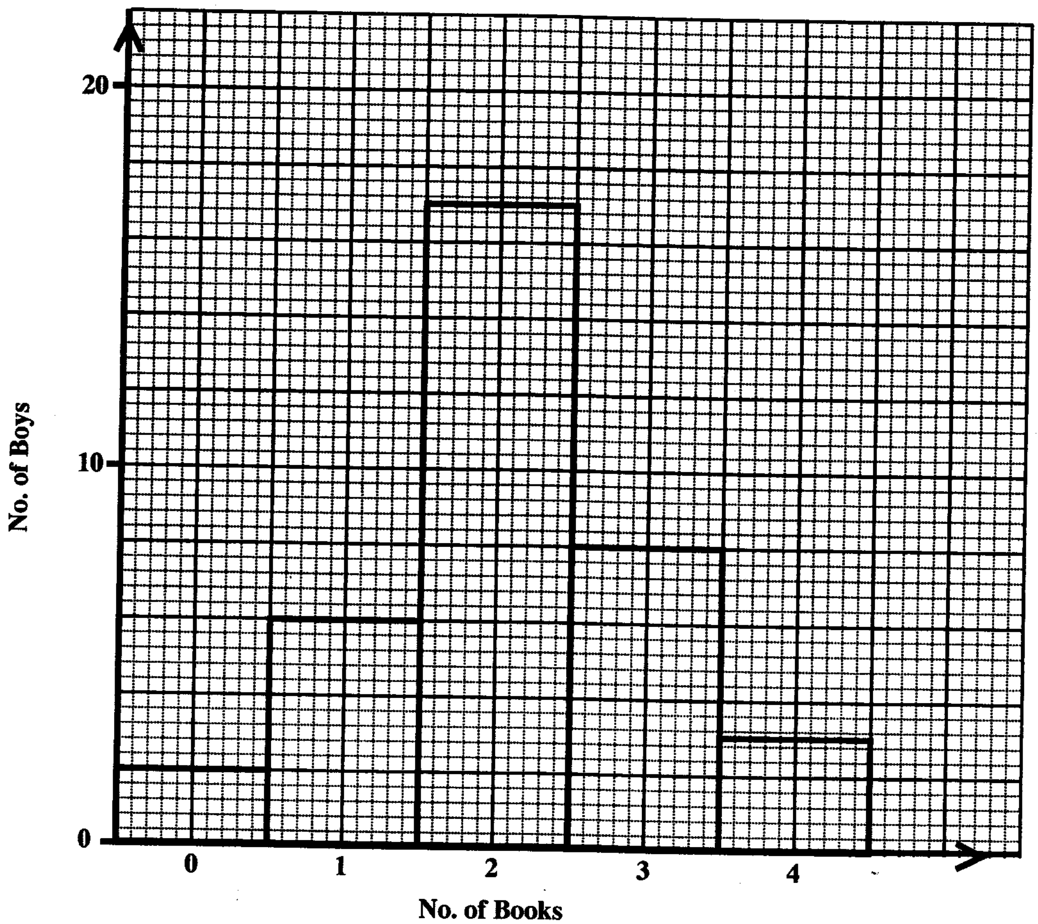
(i) the circle **(2 marks)**

(ii) the square OPQR **(2 marks)**

(iii) the shaded region. **(3 marks)**

Total 10 marks

5. In a survey, all the boys in a Book Club were asked how many books they each read during the Easter holiday. The results are shown in the bar graph below.



(a) Draw a frequency table to represent the data shown in the bar graph. (3 marks)

(b) How many boys are there in the Book Club? (2 marks)

(c) What is the modal number of books read? (1 mark)

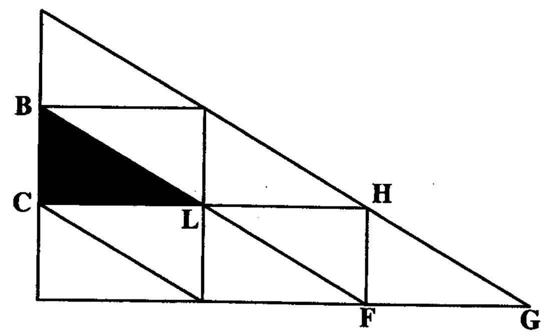
(d) How many books did the boys read during the Easter holiday? (2 marks)

(e) Calculate the mean number of books read. (2 marks)

(f) What is the probability that a boy chosen at random read THREE OR MORE books? (2 marks)

Total 12 marks

6. (a) The diagram below shows a pattern made of congruent right-angled triangles. In each triangle, the sides meeting at a right angle are 1 unit and 2 units long.



(i) Describe FULLY the single transformation that will map triangle BCL onto triangle FHL. **(3 marks)**

(ii) Describe FULLY the single transformation that will map triangle BCL onto triangle HFG. **(3 marks)**

(b) (i) **Using a ruler, a pencil and a pair of compasses**, construct parallelogram WXYZ in which

WX = 7.0 cm
WZ = 5.5 cm and
$\angle XWZ = 60^\circ$. **(5 marks)**

(ii) Measure and state the length of the diagonal WY. **(1 mark)**

Total 12 marks

7. Given that $y = x^2 - 4x$, copy and complete the table below.

(a)

x	-1	0	1	2	3	4	5
y	5		-3	-4		0	

(3 marks)

(b) **Using a scale of 2 cm to represent 1 unit on both axes**, draw the graph of the function $y = x^2 - 4x$ for $-1 \leq x \leq 5$. **(4 marks)**

(c) (i) On the same axes used in (b) above, draw the line $y = 2$. **(1 mark)**

(ii) State the ***x*-coordinates** of the two points at which the curve meets the line. **(2 marks)**

(iii) Hence, write the equation whose roots are the x-coordinates stated in (c) (ii). **(1 mark)**

Total 11 marks

8. (a) The table below shows the work done by a student in calculating the sum of the first **n** natural numbers.

Information is missing from some rows of the table. Study the pattern and complete, **in your answer booklet**, the rows marked (i) and (ii).

	n	SERIES	SUM	FORMULA	
	1	1	1	$\frac{1}{2}(1)(1+1)$	
	2	1 + 2	3	$\frac{1}{2}(2)(2+1)$	
	3	1 + 2 + 3	6	$\frac{1}{2}(3)(3+1)$	
	4	1 + 2 + 3 + 4	10	$\frac{1}{2}(4)(4+1)$	
(i)	6	1 + 2 + 3 + 4 + 5+ 6	______	______________	(**3 marks)**
	8	1 + 2 + 3 + ... + 8	36	$\frac{1}{2}(8)(8+1)$	
(ii)	n	1 + 2 + 3 + ... + n		____________	(**2 marks)**

(b) After doing additional calculations, the student stated that:

$1^3 + 2^3 + 3^3 = 36 = 6^2$ and $1^3 + 2^3 + 3^3 + 4^3 = 100 = 10^2$.

Using the pattern observed in these two statements, determine the sum of the series:

(i) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3$ (**1 mark**)

(ii) $1^3 + 2^3 + 3^3 + \ldots + n^3$. (**2 marks)**

(c) Hence, or otherwise, determine the EXACT value of the sum of the series:

$1^3 + 2^3 + 3^3 + 4^3 + \ldots + 12^3$ (**2 marks)**

Total 10 marks

SECTION II

There are SIX questions in this section.

Answer TWO questions in this section

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

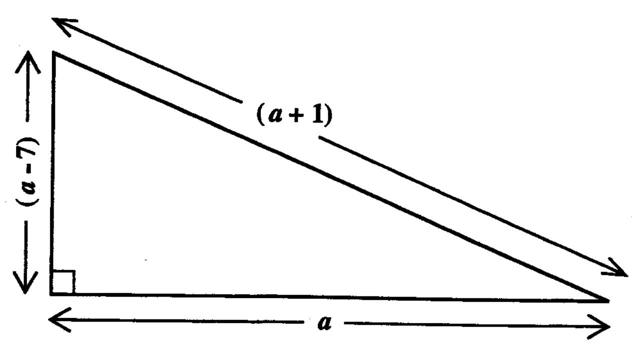
9. (a) The volume, **V**, of a gas varies inversely as the pressure **P**, when the temperature is held constant.

(i) Write an equation relating **V** and **P**. **(2 marks)**

(ii) If **V** = 12.8 when **P** = 500, determine the constant of variation. **(2 marks)**

(iii) Calculate the value of **V** when **P** = 480. **(2 marks)**

(b) The lengths, in cm, of the sides of the right-angled triangle shown below are a, (a – 7), and (a + 1).



(i) Using Pythagoras theorem, write an equation in terms of a to represent the relationship among the three sides. **(2 marks)**

(ii) Solve the equation for a. **(4 marks)**

(iii) Hence, state the lengths of the THREE sides of the triangle. **(3 marks)**

Total 15 marks

10. (a) A school buys x balls and y bats.

The total number of balls and bats is no more than 30.

(i) Write an inequality to represent this information. **(2 marks)**

The school budget allows no more than \$360 to be spent on balls and bats. The cost of a ball is \$6 and the cost of a bat is \$24.

(ii) Write an inequality to represent this information. **(2 marks)**

(b) (i) **Using a scale of 2 cm on the x-axis to represent 10 balls and 2 cm on the y-axis to represent 5 bats**, draw the graphs of the lines associated with the inequalities at (a) (i) and (ii) above. **(5 marks)**

(ii) Shade the region which satisfies the two inequalities at (a) (i) and (ii) and the inequalities $x \geq 0$ and $y \geq 0$. **(1 mark)**

(iii) Use your graph to write the coordinates of the vertices of the shaded region. **(2 marks)**

(c) The balls and bats are sold to students. The school makes a profit of \$1 on each ball and \$3 on each bat. The equation $P = x + 3y$ represents the total profit that may be collected from the sale of these items.

(i) Use the coordinates of the vertices given at (b) (iii) above to determine the profit for each of those combinations. **(2 marks)**

(ii) Hence, state the maximum profit that may be made. **(1 mark)**

Total 15 marks

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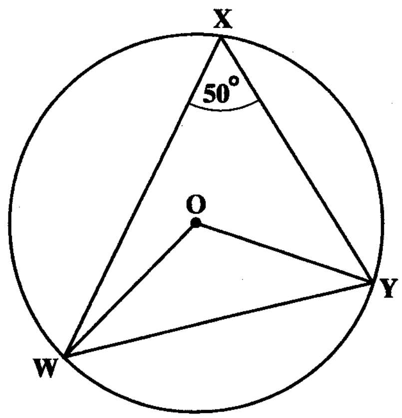
GEOMETRY AND TRIGONOMETRY

11. (a) In the diagram below, **not drawn to scale**, O is the centre of the circle WXY and $\angle WXY = 50^0$.



Calculate, giving a reason for EACH step of your answer,

(i) $\angle WOY$ **(2 marks)**

(ii) $\angle OWY$ **(2 marks)**

(b) (i) Sketch a diagram to represent the information given below.
Show clearly all measurements and any north-south lines that may be required.

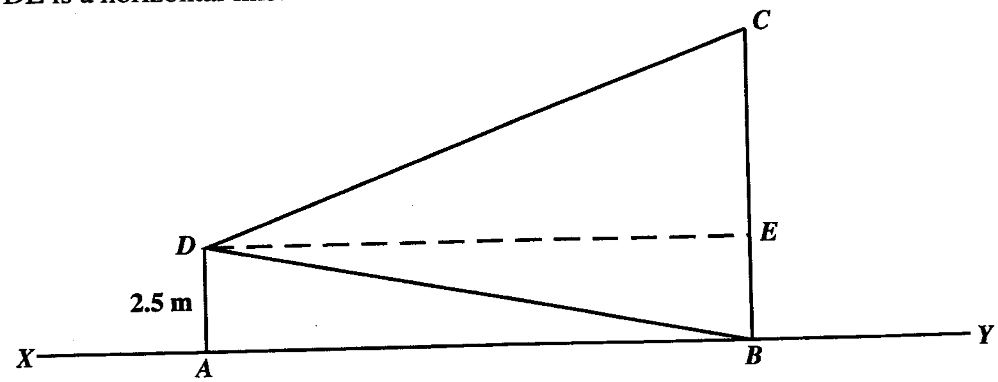
A, B and C are three buoys.
B is 125 m due east of A.
The bearing of C from B is 190^0.
CB = 75 m. **(5 marks)**

(ii) Calculate, to one decimal place, the distance AC. **(3 marks)**

(iii) Calculate, to the nearest degree, the bearing of C from A. **(3 marks)**

Total 15 marks

12. (a) The diagram below, **not drawn to scale**, shows a vertical pole, AD, and a vertical tower, BC standing on horizontal ground XABY. The height of the pole is 2.5 metres. From the point D, the angle of depression of B is 5° and the angle of elevation of C is 20°. DE is a horizontal line.



Calculate, to one decimal place

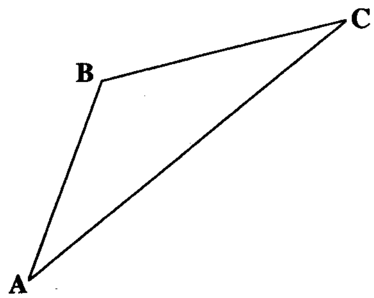
(i) the horizontal distance AB **(2 marks)**

(ii) the height of the tower, BC. **(4 marks)**

... shows a sketch of the earth with the North and

VECTORS AND MATRICES

13. (a) In triangle ABC, **not drawn to scale**, P and Q are the mid-points of AB and BC respectively.



(i) Make a sketch of the diagram and show the points P and Q . (**1 mark**)

(ii) Given that $\overrightarrow{AB} = 2\boldsymbol{x}$ and $\overrightarrow{BC} = 3\boldsymbol{y}$, write, in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$, an expression for

a) $\overrightarrow{AC}$ (**1 mark**)

b) $\overrightarrow{PQ}$ (**2 marks**)

(iii) Hence show that $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC}$ (**2 marks**)

(b) The position vectors of the points R, S and T relative to the origin are

$$\overrightarrow{OR} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad \overrightarrow{OS} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \qquad \overrightarrow{OT} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

(i) Express in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors

a) $\overrightarrow{RT}$

b) $\overrightarrow{SR}$ (**4 marks**)

(ii) a) The point F is such that $RF = FT$. Use a vector method to determine the position vector of F.

b) Hence, state the coordinates of F. (**5 marks**)

Total 15 marks

14. A, B and C are matrices such that:

(a) $A = (2 \quad 1)$, $B = \begin{pmatrix} 1 & x \\ y & -2 \end{pmatrix}$ and $C = (5 \quad 6)$

Given that AB = C, calculate the values of x and y. **(5 marks)**

(b) Given the matrix $R = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$,

(i) Show that R is non-singular. **(2 marks)**

(ii) Find R^{-1}, the inverse of R. **(2 marks)**

(iii) Show that $R R^{-1} = I$ **(2 marks)**

(iv) Using a matrix method, solve the pair of simultaneous equations

$$2x - y = 0$$
$$x + 3y = 7$$

(4 marks)

Total 15 marks

END OF TEST