JANUARY 2006

CXC

MATHEMATICS

Paper 02 - General Proficiency

2 hours 40 minutes

LIST OF FORMULAE

Volume of prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle

Area of trapezium — $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular height.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry ratios

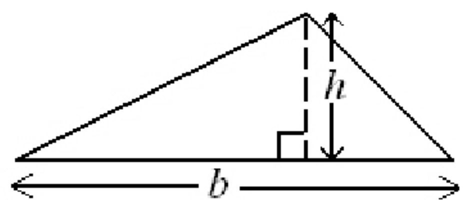
$$\sin \Theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \Theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.



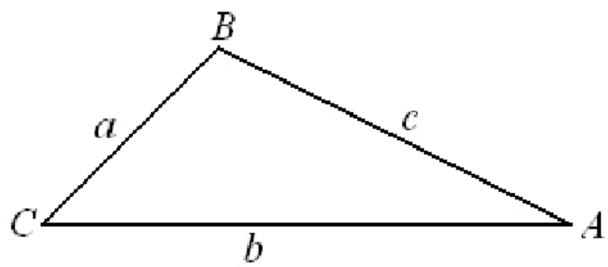
Area of $\Delta ABC = \frac{1}{2}ab \sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. Using a calculator, or otherwise, calculate

(i) the exact value of $\dfrac{2\frac{1}{4} \times \frac{4}{5}}{\frac{3}{5} - \frac{1}{2}}$

(4 marks)

(ii) correct to 3 significant figures, the value of $18.75 - (2.11)^2$.

(3 marks)

(b) A loan of $12 000 was borrowed from a bank at 14% per annum.

Calculate

(i) the interest on the loan at the end of the first year **(2 marks)**

(ii) the total amount owing at the end of the first year. **(1 mark)**

A repayment of $7 800 was made at the start of the second year.

Calculate

(iii) the amount still outstanding at the start of the second year **(1 mark)**

(iv) the interest on the outstanding amount at the end of second year. **(1 mark)**

Total 12 marks

2. (a) If a = 2, b = -3 and c = 4, evaluate

(i) $ab - bc$ **(1 mark)**

(ii) $b(a - c)^2$ **(2 mark)**

(b) Solve for x where $x \in \mathbf{Z}$:

(i) $5x + 6y = 37$ **(3 marks)**

(ii) $2x + 3y = 4$. **(3 marks)**

(c) The cost of ONE muffin is $m.

(i) Write an algebraic expression in m for the cost of:

a) FIVE muffins **(1 mark)**

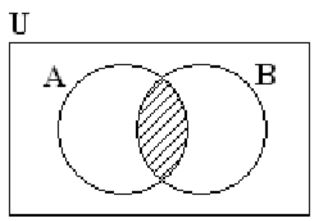
b) SIX cupcakes **(1 mark)**

(ii) Write an equation in terms of m, to represent the following information.

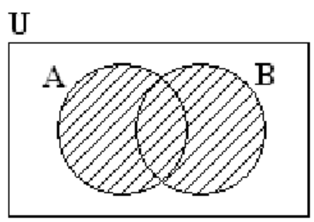
The TOTAL cost of 5 muffins and 6 cupcakes is \$31.50. **(1 mark)**

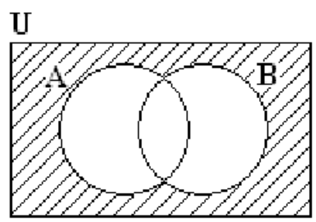
Total 12 marks

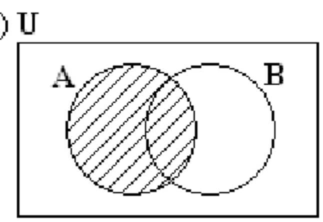
3. (a) Describe, using set notation only, the shaded region in each Venn diagram below. **The first one is done for you.**



$\underline{A \cap B}$

(i)


(ii)


(iii)


(3 marks)

(b) The following information is given.

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

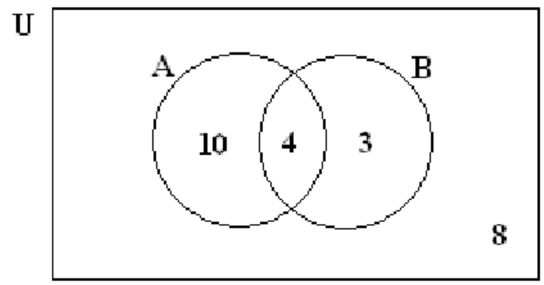
P = {prime numbers}

Q = {odd numbers}

Draw a Venn diagram to represent the information above

(3 marks)

(c) The Venn diagram below shows the number of elements in each region.



Determine how many elements are in EACH of the following sets:

(i) $A \cup B$ **(1 mark)**

(ii) $A \cap B$ **(1 mark)**

(iii) $(A \cap B)'$ **(1 mark)**

(iv) U **(1 mark)**

Total 10 marks

4. (a) (i) Using a pencil, ruler, and a pair of compasses only, construct Δ ABC with BC = 6 cm and AB = AC = 8 cm.

All construction lines must be clearly shown.

(3 marks)

(ii) Draw a line segment AD such that AD meets BC at D and is perpendicular to BC.

(2 marks)

(iii) Measure and state

a) The length of the line segment AD **(1 mark)**

b) the size of angle ABC **(1 mark)**

(b) P is the point (2, 4) and Q is the point (6, 10).

Calculate

(i) The gradient of PQ **(2 marks)**

(ii) The midpoint of PQ. **(2 marks)**

Total 11 marks

5. (a) f and g are functions defined as follows

$f: x \rightarrow 7x + 4$

$g: x \rightarrow \frac{1}{2x}$

(i) g (3)

(1 mark)

(ii) f (- 2)

(2 marks)

(iii) $f^{-1}(11)$

(2 marks)

(b) On the answer sheet, Δ ABC is mapped onto Δ A'B'C' under a reflection.

(i) Write down the equation of the mirror line.

Δ A'B'C' is mapped onto Δ A"B"C" by a rotation of 180° about the point (5, 4).

(1 mark)

(ii) Determine the coordinates of the vertices of Δ A"B"C".

(3 marks)

(iii) State the transformation that maps Δ ABC onto Δ A"B"C".

(2 marks)

Total 11 marks

6. The table below shows a frequency distribution of the scores of 100 students in an examination.

Scores	**Frequency**	**Cumulative Frequency**
21 - 25	5	5
26 - 30	18	

31 - 35	23	
36 - 40	22	
41 - 45	21	
46 - 50	11	100

(i) Copy and complete the table above to show the cumulative frequency for the distribution.

(2 marks)

(ii) Using a scale of 2 cm to represent a score of 5 on the horizontal axis and a scale of 2 cm to represent 10 students on the vertical axis, draw a cumulative frequency curve of the scores. Start your horizontal scale at 20.

(6 marks)

(iii) Using the cumulative frequency curve, determine the median score for the distribution.

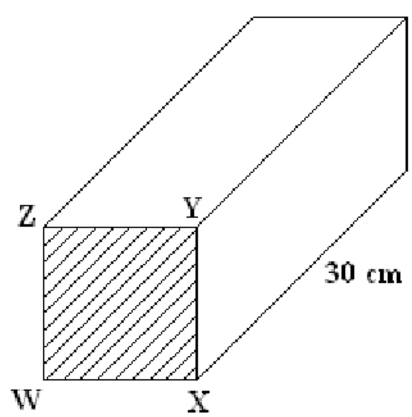
(2 marks)

(iv) What is the probability that s student chosen at random has a score greater than 40?

(2 marks)

Total 12 marks

7. (a) The diagram below, **not drawn to scale**, shows a prism of length 30 cm. The cross-section WXYZ is a square with area 144 cm².



Calculate

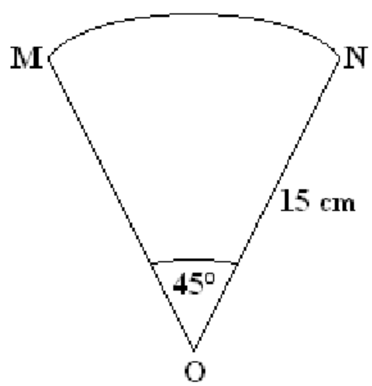
(i) the volume, in cm³, of the prism

(2 marks)

(ii) the total surface area, in cm^2, of the prism.

(2 marks)

(b) The diagram below, **not drawn to scale**, shows the sector of a circle with centre O.



Calculate, giving your answer correct to 2 decimal places

(i) the length of the minor arc MN

(2 marks)

(ii) the perimetre of the figure MON

(2 marks)

(iii) the area of the figure MON.

Total 12 marks

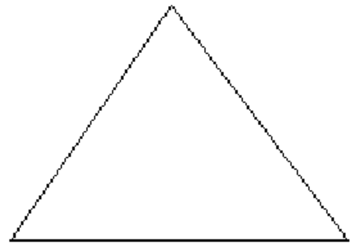
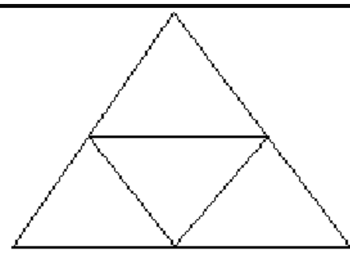
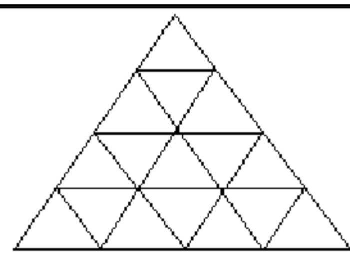
8. A large equilateral triangle is subdivided into a set of smaller equilateral triangles by the following procedure:

The midpoints of the sides of each equilateral triangle are joined to form a new set of smaller triangles.

The procedure is repeated many times.

The table below shows the results when the above procedure has been repeated twice, that is, when n = 2.

n	Result after each step	No. of triangles formed

0		1
1		4
2		16
3		(i)
6		(ii)
(iii)		65536
m		(iv)

(i) Calculate the number of triangles formed when n = 3.

(2 marks)

(ii) Determine the number of triangles formed when n = 6.

(2 marks)

A shape has 65 536 small triangles.

(iii) Calculate the value of n.

(3 marks)

(iv) Determine the number of small triangles in a shape after carrying out the procedure m times.

(3 marks)

Total 10 marks

SECTION II

Answer TWO questions in this section

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Factorize completely

(i) $2p^2 - 7p + 3$ **(1 mark)**

(ii) $5p + 5q + p^2 - q^2$ **(2 marks)**

(b) Expand $(x + 3)^2 (x - 4)$, writing your answer in descending powers of x.

(3 marks)

(c) Given $f(x) = 2x^2 + 4x - 5$

(i) write f(x) in the form $f(x) = a(x+b)^2 + c$ where $a,b,c \in$ **R**

(3 marks)

(ii) state the equation of the axis of symmetry

(1 mark)

(iii) state the coordinates of the minimum point

(1 mark)

(iv) sketch the graph of f(x)

(2 marks)

(v) on the graph of f(x) show clearly

a) the minimum point **(1 marks)**

b) the axis of symmetry **(1 marks)**

Total 15 marks

10. Pam visits the stationery store where she intends to buy x pens and y pencils.

(a) Pam must buy at least 3 pens.

(i) Write an inequality to represent this information

(1 mark)

The TOTAL number of pens and pencils must NOT be more than 10.

(ii) Write an inequality to represent this information.

(2 marks)

EACH pen costs \$5.00 and EACH pencil costs \$2.00. more information about the pens and pencils is represented by:

$$5x + 2y \leq 35$$

(iii) Write the information represented by this inequality as asentence in your own words.

(2 marks)

(b) (i) On the answer sheet draw the graph of the TWO inequalities obtained in (a) (i) and (a) (ii) above.

(3 marks)

(ii) Write the coordinates of the vertices of the region that satisfies the four inequalities (including $y \geq 0$)

(2 marks)

(c) Pam sells the x pens and y pencils and makes a profit of \$1.50 on EACH pen and \$1.00 on EACH pencil

(i) Write an expression in x and y to represent the profit Pam makes.

(1 mark)

(ii) Calculate the MAXIMUM profit Pam makes.

(2 marks)

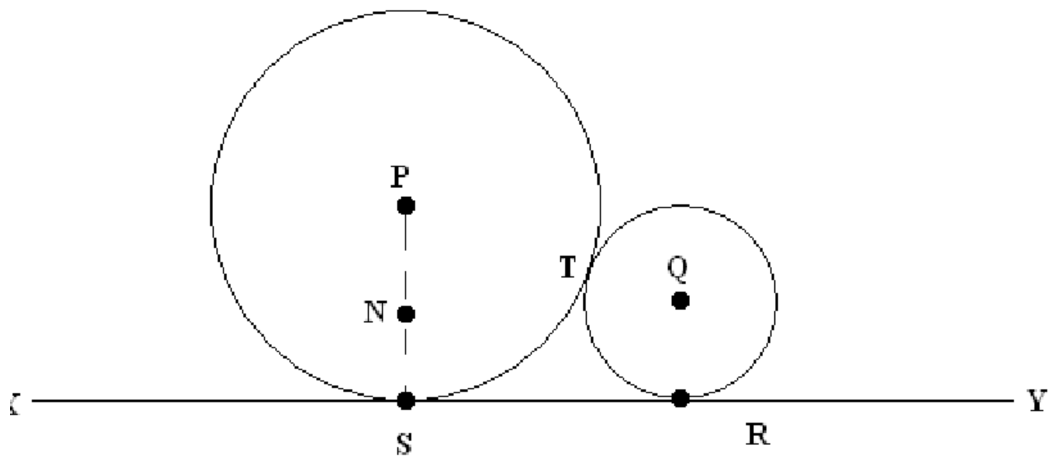
(iii) If Pam buys 4 pens, show **on your graph** the maximum number of pencils she can buy.

(2 mark)

Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a) Two circles with centres P and Q and radius 5 cm and 2 cm respectively are drawn so that they touch each other at T and a straight line XY at S and R.



(i) State, with a reason,

a) why PTQ is a straight line **(2 marks)**

b) The length PQ **(2 marks)**

c) Why PS is parallel to QR **(2 marks)**

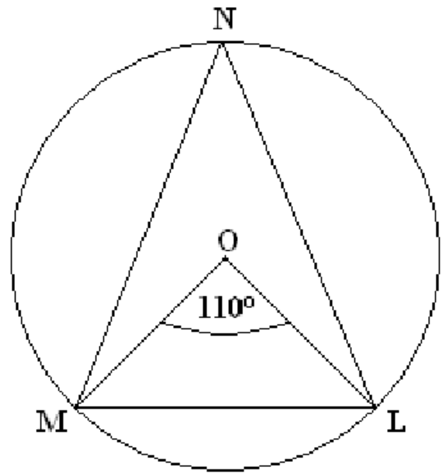
(ii) N is a point on PS such that QN is perpendicular to PS.

Calculate

a) the length PN **(2 marks)**

b) the length RS **(2 marks)**

(b) In the diagram below, **not drawn to scale,** O is the centre of the circle. The measure of angle LOM is 110°.



Calculate, giving reasons for your answers, the size of each of the following angles

(i) <MNL

(2 marks)

(ii) <LMO

(3 marks)

Total 15 marks

12. A boat leaves a dock at point A and travels for a distance of 15 km to poi B on a bearing of 135°.

The boat then changes course and travels for a distance of 8 km to point c on a bearing of 060°.

(a) Illustrate the above information in a clearly labelled diagram.

(2 marks)

The diagram should show the

(i) north direction **(1 marks)**

(ii) bearings 135° and 060° **(2 marks)**

(iii) distances 8 km and 15 km. **(2 marks)**

(b) Calculate

(i) the distance AC

(3 marks)

(ii) <BCA

(3 marks)

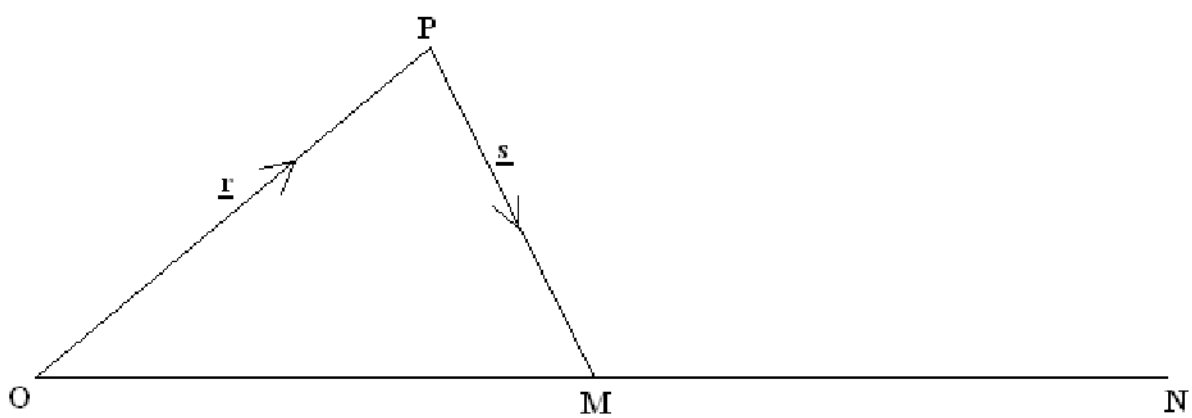
(iii) the bearing of A from C

(2 mark)

Total 15 marks

VECTORS AND MATRICES

13. In the diagram below, M is the midpoint of $\overrightarrow{ON}$.



(a) (i) Sketch the diagram above in your answer booklet and insert the point X on $\overrightarrow{OM}$.

such that $\overrightarrow{OX} = \frac{1}{3}\overrightarrow{OM}$.

(1 mark)

(ii) Produce PX to Q such that $\overrightarrow{PX} = 4\overrightarrow{XQ}$.

(1 mark)

(b) Writhe the following in terms of **r** and **s**.

(i) $\overrightarrow{OM}$

(2 marks)

(ii) $\overrightarrow{PX}$

(3 marks)

(iii) $\overrightarrow{QM}$

(4 marks)

(c) Show that $\overrightarrow{PN} = 2\overrightarrow{PM} + \overrightarrow{OP}$

(4 marks)

Total 15 marks

14. (a) Given that $D = \begin{pmatrix} 1 & 9p \\ p & 4 \end{pmatrix}$ is a singular matrix, determine the value(s) of p.

(4 marks)

(b) Given the linear equations

$2x + 5y = 6$

$3x + 4y = 8$

(i) Write the equations in the form AX = B where A, X and B are matrices.

(2 marks)

(ii) a) Calculate the determinant of the matrix A.

(2 marks)

b) Show that $A^{-1} = \begin{pmatrix} \frac{-4}{7} & \frac{5}{7} \\ \frac{3}{7} & \frac{-2}{7} \end{pmatrix}$.

(3 marks)

c) Use the matrix A^{-1} to solve for x and y.

(5 marks)

Total 15 marks

END OF TEST
