FORM TP 2017085

TEST CODE **01234020**

MAY/JUNE 2017

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has EIGHT questions and Section II has THREE questions.
3. Answer ALL questions in Section I, and any TWO questions from Section II.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. All working MUST be clearly shown.
7. **A list of formulae is provided on page 4 of this booklet.**
8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
9. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

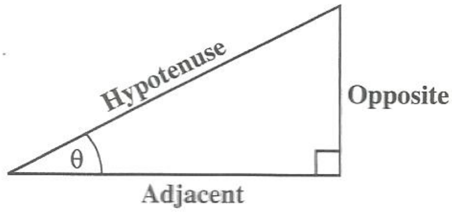
Volume of a prism	$V = Ah$ where A is the area of the cross-section and h is the perpendicular length.
Volume of cylinder	$V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.
Volume of a right pyramid	$V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.
Circumference	$C = 2\pi r$ where r is the radius of the circle.
Arc length	$S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.
Area of a circle	$A = \pi r^2$ where r is the radius of the circle.
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.
Area of trapezium	$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.
Roots of quadratic equations	If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

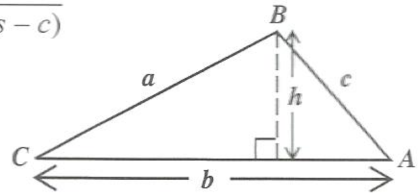
$$\text{Area of } \Delta ABC = \frac{1}{2}ab\sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

(i) $\left(4\frac{1}{3} - 1\frac{2}{5}\right) \div \frac{4}{15}$

(2 marks)

(ii) $\frac{(3.1 - 1.15)^2}{0.005}$

(2 marks)

(b) A store is promoting a new mobile phone under two plans: Plan A and Plan B.

The plans are advertised as shown in the table below.

	Plan A	**Plan B**
Deposit	$400	$600
Monthly instalment	$65	$80
Number of months to repay	12	6
Tax on ALL payments	0%	5%

(i) Calculate the TOTAL cost of a phone under Plan A.

(2 marks)

(ii) Determine which of the two plans, A or B, is the better deal. **Justify your answer.**

(2 marks)

(c) John's monthly electricity bill is based on the number of kWh of electricity that he consumes for that month. He is charged $5.10 per kWh of electricity consumed. For the month of March 2016, two meter readings are displayed in the table below.

	Meter Readings (kWh)
Beginning 01 March	0 3 0 1 1
Ending 31 March	0 3 3 0 7

(i) Calculate the TOTAL amount that John pays for electricity consumption for the month of March 2016.

(2 marks)

(ii) For the next month, April 2016, John pays $2351.10 for electricity consumption. Determine his meter reading at the end of April 2016.

(2 marks)

Total 12 marks

2. (a) Factorize the following expressions completely.

(i) $6y^2 - 18xy$

(2 marks)

(ii) $4m^2 - 1$

(1 mark)

(iii) $2t^2 - 3t - 2$

(2 marks)

(b) Write as a single fraction and simplify

$$\frac{5p+2}{3} - \frac{3p-1}{4}.$$

(2 marks)

(c) A formula is given as $d = \sqrt{\frac{4h}{5}}$.

(i) Determine the value of d when $h = 29$. Give your answer correct to 3 significant figures.

(2 marks)

(ii) Make h the subject of the formula.

(2 marks)

Total 11 marks

3. (a) The universal set U is defined as follows:

$$U = \{x : x \in N, 2 < x < 12\}.$$

The sets M and R are subsets of U such that

$M = \{\text{odd numbers}\}$
$R = \{\text{square numbers}\}$.

(i) List the members of the subset M.

....

....

(1 mark)

(ii) List the members of the subset R.

....

....

(1 mark)

(iii) Draw a Venn diagram that represents the relationship among the defined subsets of U.

(4 marks)

(b) Using **a ruler, a pencil and a pair of compasses,**

(i) construct accurately, the square *ABCD*, with sides 6 cm **(3 marks)**

(ii) construct, as an extension of your drawing in (b) (i), the trapezium *DABQ* so that $\angle ABQ = 120°$. **(2 marks)**

[Note: Credit will be given for clearly drawn construction lines.]

(iii) Hence, measure and state the length of *BQ*.

....

(1 mark)

Total 12 marks

4. (a) The function f is defined as $f(x) = \frac{1}{3}x - 2$.

(i) Find the value of $f(3) + f(-3)$.

(2 marks)

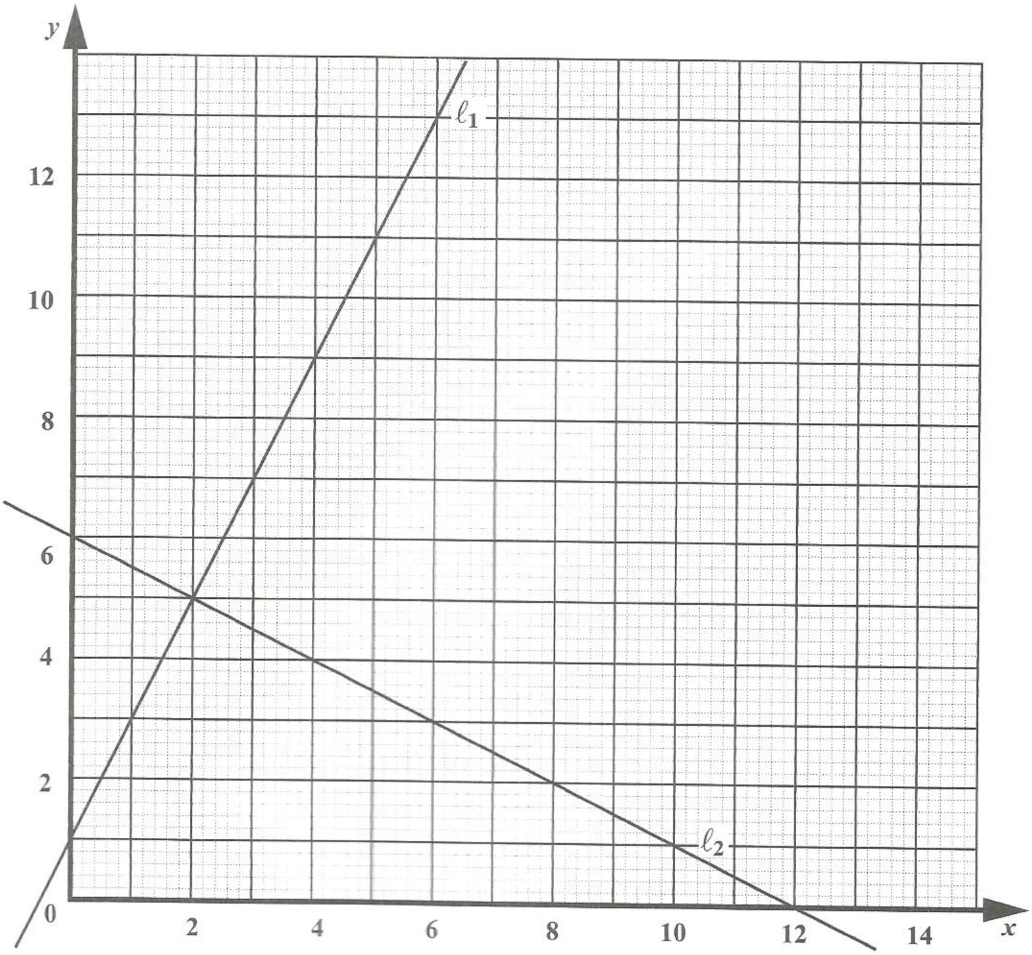
(ii) Calculate the value of x for which $f(x) = 5$.

(2 marks)

(iii) Determine the inverse function $f^{-1}(x)$.

(2 marks)

(b) The graph below shows two straight lines, ℓ_1 and ℓ_2. Line ℓ_1 intercepts the y-axis at (0, 1). Line ℓ_2 intercepts the x and y axes at (12, 0) and (0, 6) respectively.



(i) Calculate the gradient of the lines ℓ_1 and ℓ_2.

(2 marks)

(ii) Determine the equation of the line ℓ_1.

(2 marks)

(iii) What is the relationship between ℓ_1 and ℓ_2? Give a reason for your answer.

....

....

....

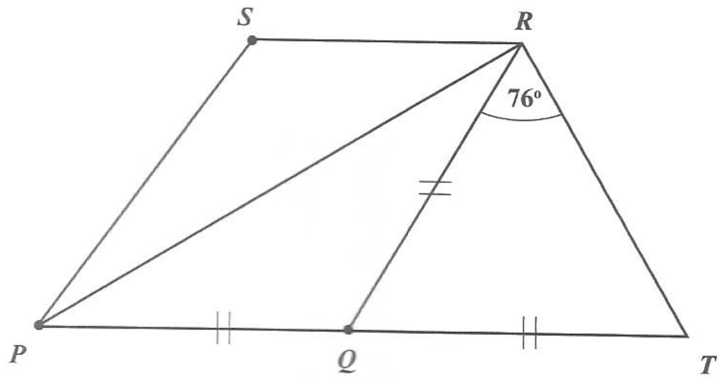
....

....

(2 marks)

Total 12 marks

5. (a) *PTRS*, **not drawn to scale**, is a quadrilateral. Q is a point on PT such that $QT = QR = QP$. Angle $QRT = 76°$.



Determine, giving a reason for each step of your answer, the measure of

(i) angle RQT

(2 marks)

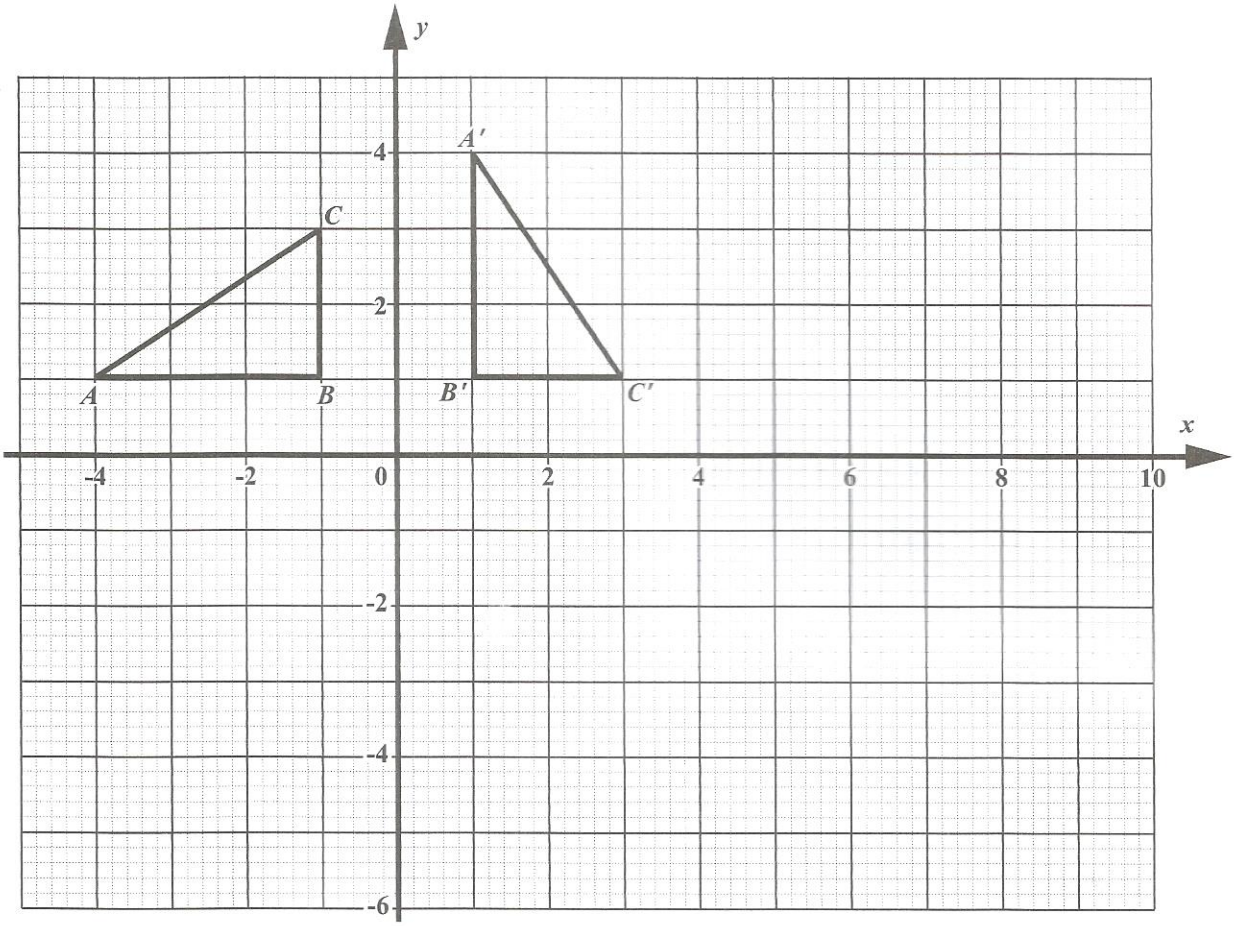
(ii) angle PRT

(2 marks)

(iii) angle SPT, given that angle $SRT = 145°$ and angle $PSR = 100°$.

(2 marks)

(b) The diagram below shows triangle ABC and its image, $A'B'C'$, under a single transformation.



(i) Describe **completely** the transformation that maps $\Delta\, ABC$ to $\Delta\, A'B'C'$.

....

....

....

....

....

(3 marks)

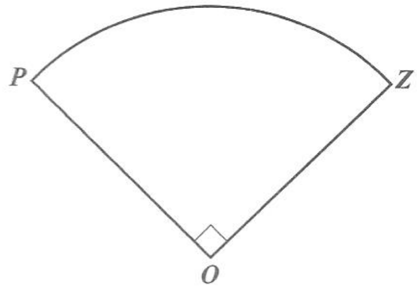
(ii) The translation vector $T = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ maps $\Delta\, A'B'C'$ to $\Delta\, A''B''C''$.

On the diagram above, draw the $\Delta\, A''B''C''$. **(2 marks)**

Total 11 marks

6. (a) **In this problem take π to be $\frac{22}{7}$.**

The diagram below, **not drawn to scale**, shows a field in the shape of a sector of a circle, with centre O and diameter 28 m. Angle POZ is 90°.



Calculate

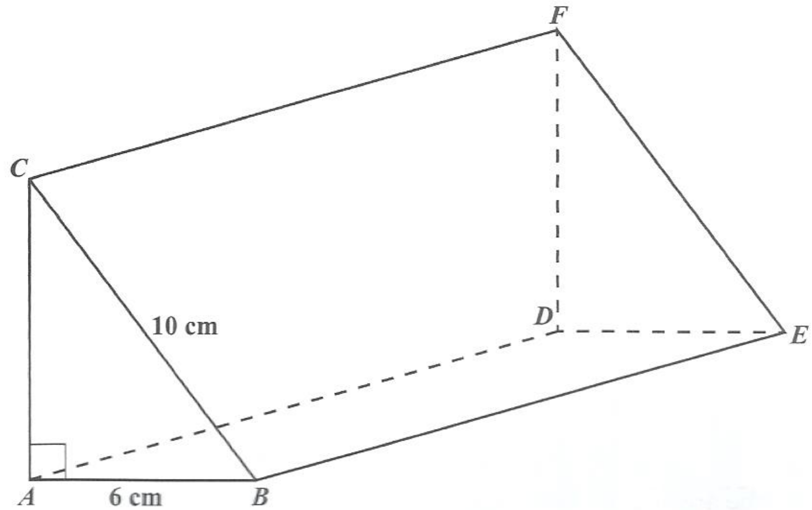
(i) the area of the field

(2 marks)

(ii) the perimeter of the field.

(3 marks)

(b) The diagram below, **not drawn to scale**, shows a triangular prism $ABCDEF$. The cross-section is the right-angled triangle, ABC, where $AB = 6$ cm and $BC = 10$ cm.



Calculate

(i) the area of the triangle ABC

(2 marks)

(ii) the length of the prism, if the volume is 540 cm^3

(2 marks)

(iii) the surface area of the prism.

(2 marks)

Total 11 marks

7. The table below shows the speeds, to the nearest **kmh^{-1}**, of 90 vehicles that pass a checkpoint.

Speed (in kmh^{-1})	Frequency	Cumulative Frequency
0–19	5	5
20–39	11	16
40–59	26	
60–79	37	
80–99	9	
100–119	2	

(a) For the class interval 20–39, as written in the table above, complete the following sentences.

(i) The upper class limit is ....

(1 mark)

(ii) The class width is ....

(1 mark)

(iii) Sixteen vehicles passed a checkpoint at no more than .... kmh^{-1}.

(1 mark)

(b) Complete the table shown above by inserting the missing values for the cumulative frequency column. **(2 marks)**

(c) On the grid provided **on page 21**, using a scale of 2 cm to represent 20 kmh^{-1} on the x-axis, and 2 cm to represent 10 vehicles on the y-axis, draw the cumulative frequency curve to represent the information in the table. **(4 marks)**

(d) (i) On your graph, draw reference lines to estimate the speed at which no more than 50% of the vehicles drove as they passed the checkpoint. **(1 mark)**

(ii) What is the estimated speed?

....

(1 mark)

Total 11 marks

8. The first four figures in a sequence are shown below. Figure 1 is a single black dot, while each of the others consist of black dots arranged in an equilateral manner.

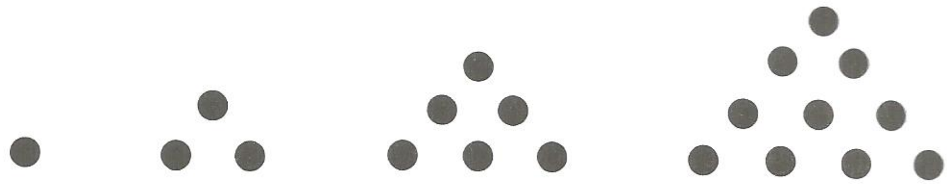


Figure 1 **Figure 2** **Figure 3** **Figure 4**

(a) Draw Figure 5 of the sequence in the space below.

(2 marks)

(b) How many dots would be in Figure 6?

....

(1 mark)

The table below refers to the figures and the number of dots in each figure. Study the patterns shown.

Figure, n	**Number of Dots, d, in terms of n**	**Number of Dots Used, d**
1	$\frac{1}{2} \times 1 \times (1 + 1)$	1
2	$\frac{1}{2} \times 2 \times (2 + 1)$	3
3	$\frac{1}{2} \times 3 \times (3 + 1)$	6
⋮		
11		
⋮		
n		

(c) Complete the row which corresponds to Figure 11 in the table above. **(2 marks)**

(d) Determine which figure in the sequence has 210 dots.

(2 marks)

(e) Write a simplified algebraic expression for the number of dots, d, in the Figure n.

(1 mark)

(f) Show that there is no diagram that has exactly 1000 dots.

(2 marks)

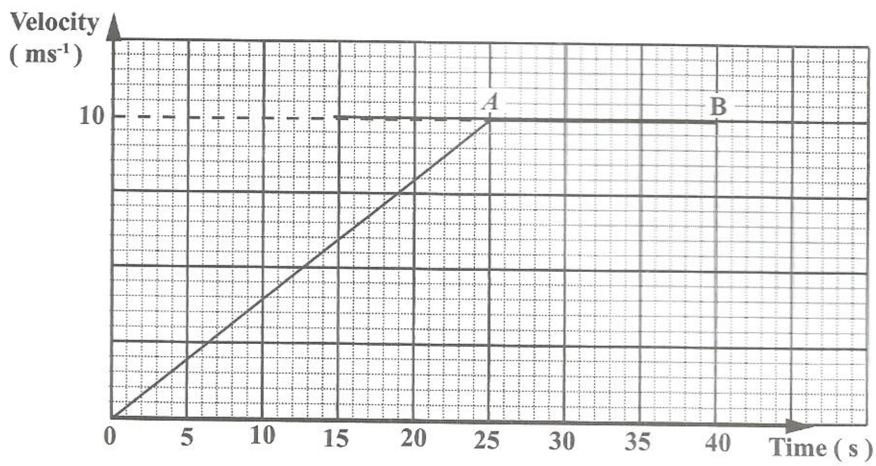
Total 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The velocity–time graph below shows the motion of a cyclist over a period of 40 seconds.



(i) Calculate the gradient of

a) OA

(1 mark)

b) AB.

(1 mark)

(ii) Complete the following statements.

The cyclist started from rest, where his velocity was .... ms^{-1}, and steadily increased his velocity by .... ms^{-1} each second during the first 25 seconds.

During the next 15 seconds, his velocity remained constant, that is, his acceleration was .... ms^{-2}. **(3 marks)**

(iii) Determine the average speed of the cyclist over the 40-second period.

(3 marks)

(b) Consider the following pair of simultaneous equations:

$$x^2 + 2xy = 5$$
$$x + y = 3$$

(i) WITHOUT solving, show that (1, 2) is a solution for the pair of simultaneous equations.

(2 marks)

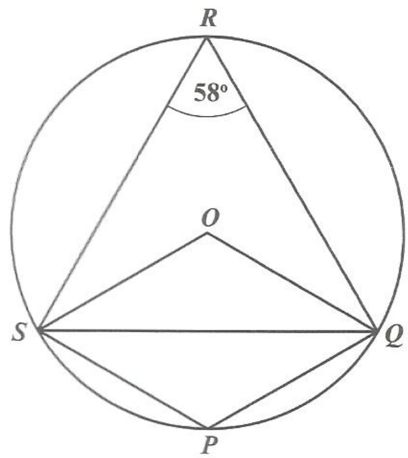
(ii) Solve the pair of simultaneous equations above to determine the **other** solution.

(5 marks)

Total 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) P, Q, R and S are four points on the circumference of the circle shown below. Angle $QRS = 58°$.



Using the geometrical properties of a circle to give reasons for each step of your answer, determine the measure of

(i) $\angle SPQ$

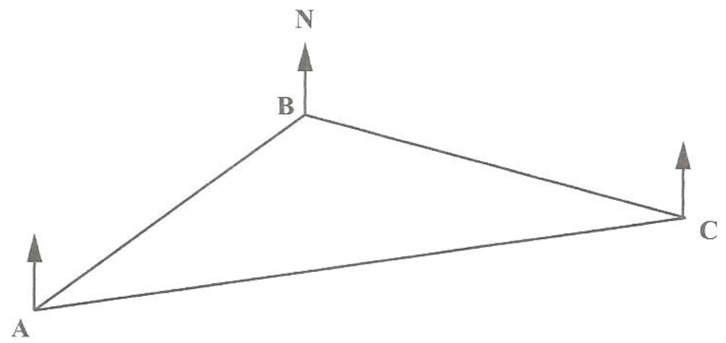
(2 marks)

(ii) $\angle OQS$

(3 marks)

(b) A ship leaves Port A and sails 52 km on a bearing of 044° to Port B. The ship then changes course to sail to Port C, 72 km away, on a bearing of 105°.

(i) On the diagram below, **not drawn to scale**, label the known distances travelled and the known angles.



(2 marks)

(ii) Determine the measure of $\angle ABC$.

(2 marks)

(iii) Calculate, to the nearest km, the distance AC.

(3 marks)

(iv) Show that the bearing of A from C, to the nearest degree, is 260°.

(3 marks)

Total 15 marks

VECTORS AND MATRICES

11. (a) Matrices A and B are such that

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & 0 \\ 3 & -1 \end{pmatrix}.$$

(i) Show by multiplying A and B, that $AB \neq BA$.

(2 marks)

(ii) Find A^{-1}, the inverse of A.

(2 marks)

(iii) Write down the 2×2 matrix representing the matrix product AA^{-1}.

(1 mark)

(b) (i) Write the following pair of simultaneous equations as a matrix equation.

$$3x + 2y = 1$$
$$5x + 4y = 5$$

(1 mark)

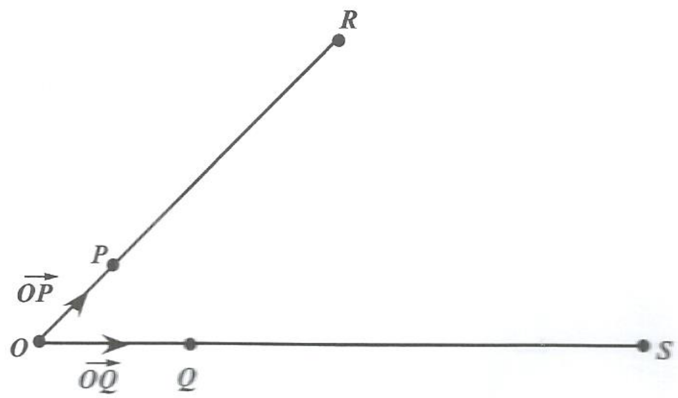
(ii) Write the solution of your matrix equation in (b) (i) as a product of two matrices.

(2 marks)

(c) The position vectors of the points P and Q relative to an origin, O, are

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \overrightarrow{OQ} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \text{ respectively.}$$

The diagram below shows that $PR = 3\ OP$ and $QS = 3\ OQ$.



(i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, vector

- $\overrightarrow{OS}$

(1 mark)

- $\overrightarrow{PQ}$

(2 marks)

- $\overrightarrow{RS}$.

(2 marks)

(ii) State TWO geometrical relationships between PQ and RS.

....

....

....

....

(2 marks)

Total 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.